B O S Z

- Boussinesq Ocean & Surf Zone model -

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TOHOKU UNIVERSITY

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CORE OCEAN AND RESOURCES ENGINEERING since 1966 AT THE UNIVERSITY OF HAWAI'I AT MANOA

FRANZIUS-INSTITUTE for Hydraulic, Estuarine and Coastal Engineering

B O S Z

in a nutshell

- Developed for energetic breaking waves

- Applications in: Flood hazard mapping,
  Disaster management,
  Studying nearshore processes
  Coastal engineering design.

- Requirements: Cover wide range of wave scenarios,
  Numerical robustness,
  Efficient computation.

As complex as necessary, as lean as possible!

BOSZ
Key Features

- Depth-integrated Boussinesq-type Equations
  - Conservative form of Nwogu's (1993) Boussinesq equations, see: Roeber & Cheung (2012)
  - Shallow water equations as subset
  - Extension with nonlinear dispersion

- Momentum Conservation
  - Imbedded conservation laws for sub- and supercritical flows

- Finite Volume - Finite Difference scheme
  - Explicit 2nd or 4th order Runge-Kutta time integration, adaptive time step,
  - Riemann solver for wave breaking

- Parallelization
  - Full OpenMP parallelization to handle large flow problems

BOSZ
Governing Equations

Conserved variables:

$$(Hu)_t + \left(Hu^2\right)_x + (Huv)_y = u\left(H_t + (Hu)_x + (Hv)_y\right) + H\left(u_t + uu_x + vu_y\right)$$

$$(Hv)_t + \left(Hv^2\right)_y + (Huv)_x = v\left(H_t + (Hv)_y + (Hu)_x\right) + H\left(v_t + vv_y + uv_x\right)$$

Continuity:

$$H_t + (Hu)_x + (Hv)_y + \left[\left(\frac{z_\alpha^2}{2} - \frac{h^2}{6}\right)h\left(u_{xx} + v_{xy}\right) + \left(z_\alpha + \frac{h}{2}\right)h\left((hu)_{xx} + (hv)_{xy}\right)\right]_x + \left[\left(\frac{z_\alpha^2}{2} - \frac{h^2}{6}\right)h\left(u_{xy} + v_{yy}\right) + \left(z_\alpha + \frac{h}{2}\right)h\left((hu)_{xy} + (hv)_{yy}\right)\right]_y = 0$$

local flux dispersion

Momentum x:

$$(Hu)_t + \left(Hu^2\right)_x + (Huv)_y + gH\eta_x + H\frac{z_\alpha^2}{2}\left[u_{xxt} + v_{xyt}\right] + Hz_\alpha\left[(hu_t)_{xx} + (hv_t)_{xy}\right] + u\left[\left(\frac{z_\alpha^2}{2} - \frac{h^2}{6}\right)h\left(u_{xx} + v_{xy}\right) + \left(z_\alpha + \frac{h}{2}\right)h\left((hu)_{xx} + (hv)_{xy}\right)\right]_x + u\left[\left(\frac{z_\alpha^2}{2} - \frac{h^2}{6}\right)h\left(u_{xy} + v_{yy}\right) + \left(z_\alpha + \frac{h}{2}\right)h\left((hu)_{xy} + (hv)_{yy}\right)\right]_y = 0$$

local flux + bottom slope dispersion

BOSZ
Equation Restructuring

Vector form: $\mathrm{U}_t + \mathrm{F}(\mathrm{U})_x + \mathrm{G}(\mathrm{U})_y + \mathrm{S}(\mathrm{U}) = 0$ **U** = [*H*, *P*, *Q*]^T

$$\mathrm{F} = \begin{bmatrix} Hu \\ Hu^2 + \frac{1}{2}g\eta^2 + g\eta h \\ Huv \end{bmatrix} \quad \mathrm{G} = \begin{bmatrix} Hv \\ Hvu \\ Hv^2 + \frac{1}{2}g\eta^2 + g\eta h \end{bmatrix} \quad \mathrm{S} = \begin{bmatrix} \psi_C \\ -g\eta h_x - H_t\psi_P + u\psi_C + \tau_1 \\ -g\eta h_y - H_t\psi_Q + v\psi_C + \tau_2 \end{bmatrix}$$

Nonlinear Shallow-Water Equations
With 2D TVD reconstruction and
HLLC Riemann solver

Dispersion terms only
containing spatial derivatives
In Finite Difference

- 2D 5th order TVD reconstruction (Kim & Kim 2005)

- Wet/dry boundary based on Audusse et al., 2004, and Liang, 2010

- Frictional drag based on Manning's *n*

- Indexing of <u>only</u> *wet* cells to save computation time

Equation Solution

Runge-Kutta time integration of 2nd or 4th order with adaptive time step

$$H'^{n+1}_{i,j} = H^n_{i,j} + \Delta t^n \left(E^n_{1i,j} + E^n_{2i,j} \right)$$

$$P'^{n+1}_{i,j} = P^n_{i,j} + \Delta t^n \left(F^n_{1i,j} + F^n_{2i,j} \right)$$

$$Q'^{n+1}_{i,j} = Q^n_{i,j} + \Delta t^n \left(G^n_{1i,j} + G^n_{2i,j} \right)$$

$$H^{n+1}_{i,j} = \frac{1}{2}\left(H'^{n+1}_{i,j} + H^n_{i,j} \right) + \frac{\Delta t^n}{2}\left(E'^{n+1}_{1i,j} + E'^{n+1}_{2i,j} \right)$$

$$P^{n+1}_{i,j} = \frac{1}{2}\left(P'^{n+1}_{i,j} + P^n_{i,j} \right) + \frac{\Delta t^n}{2}\left(F'^{n+1}_{1i,j} + F'^{n+1}_{2i,j} \right)$$

$$Q^{n+1}_{i,j} = \frac{1}{2}\left(Q'^{n+1}_{i,j} + Q^n_{i,j} \right) + \frac{\Delta t^n}{2}\left(G'^{n+1}_{1i,j} + G'^{n+1}_{2i,j} \right)$$

$$P = Hu + Hz_\alpha \left[0.5 z_\alpha \left(u_{xx} + v_{xy} \right) + \left(hu \right)_{xx} + \left(hv \right)_{xy} \right]$$

$$Q = Hv + Hz_\alpha \left[0.5 z_\alpha \left(v_{yy} + u_{xy} \right) + \left(hv \right)_{yy} + \left(hu \right)_{xy} \right]$$

Flow velocities *U* and *V* from series of one-dimensional systems of equations

Favorable to *parallelization* in the x and the y direction without data dependency

Temporal derivatives in cross-terms are evaluated with one-sided derivatives

$$\phi_P = Hz_\alpha \left[0.5 z_\alpha \left(v_{txy} \right) + \left(hv_t \right)_{xy} \right]$$

$$\phi_Q = Hz_\alpha \left[0.5 z_\alpha \left(u_{txy} \right) + \left(hu_t \right)_{xy} \right]$$

Velocity Variables

Horizontal velocity at any depth:

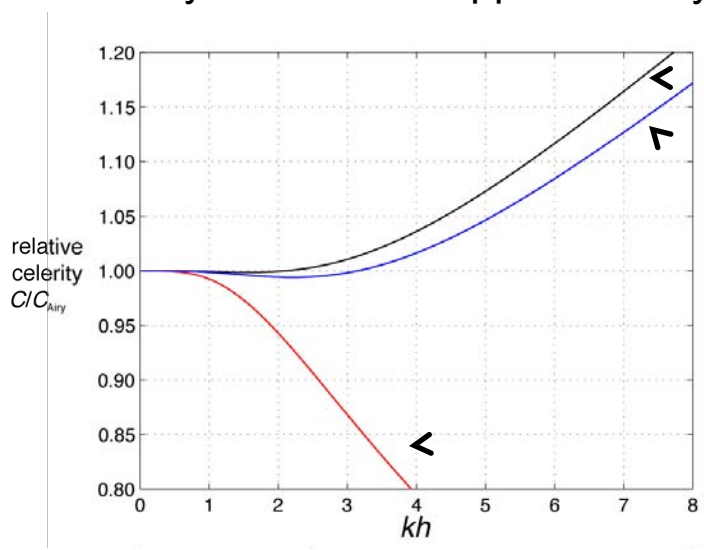
$$\mathrm{u}(\mathrm{x},z,t) = z_\alpha\left[\nabla h\left(\nabla\cdot\mathrm{u}_\alpha\right)+\nabla\left(\nabla h\cdot\mathrm{u}_\alpha\right)\right]+\frac{1}{2}z_\alpha^2\left[\nabla\left(\nabla\cdot\mathrm{u}_\alpha\right)\right]$$

Vertical velocity at any depth:

$$w(\mathrm{x},z,t) = -\left[\mathrm{u}_\alpha\cdot\nabla h+(z+h)\nabla\cdot\mathrm{u}_\alpha\right]$$

For range $-h < z < \eta$

Evaluation of velocity variables at approximately mid depth

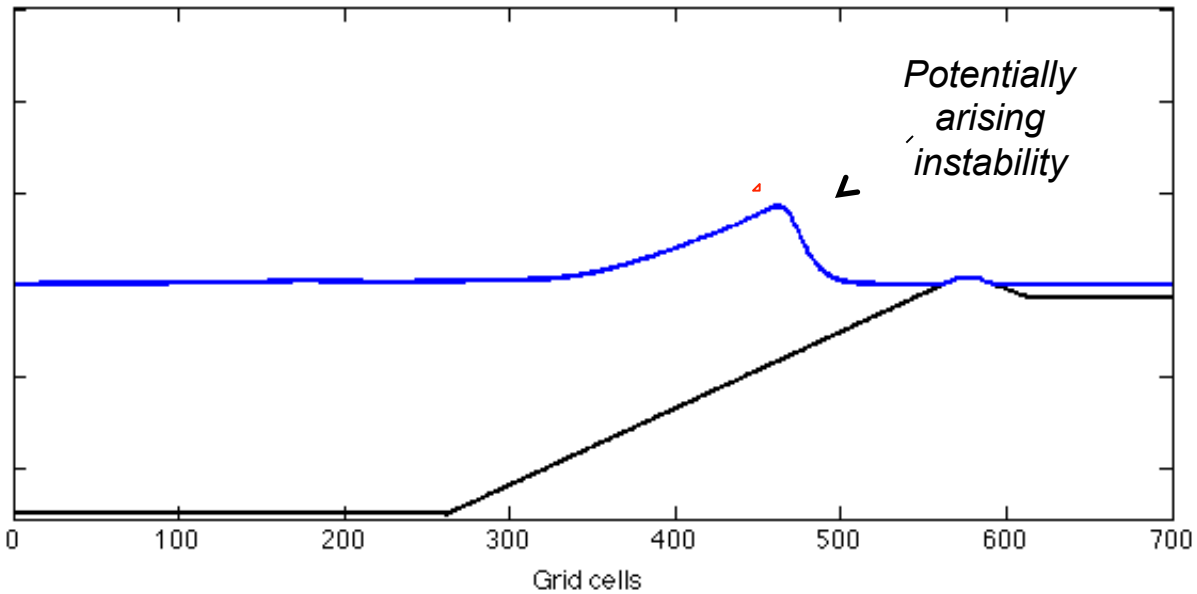


$z_\alpha = -0.53h$

$z_\alpha = -0.5208h$

$z_\alpha = -0.333h$

Wave Breaking in BOSZ

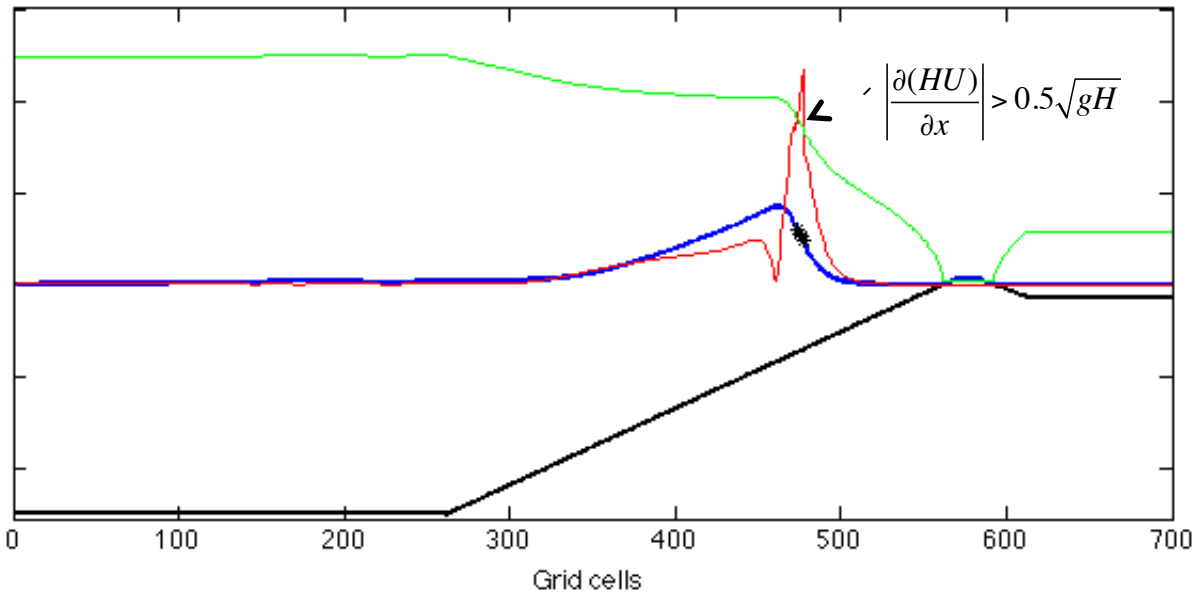


Momentum conserved for sub- and supercritical flow without additional treatment

<u>Two strategies:</u> 1. Problem Buffering - > Eddy viscosity

2. Problem Prevention - > Neglecting dispersion

Wave Breaking in BOSZ



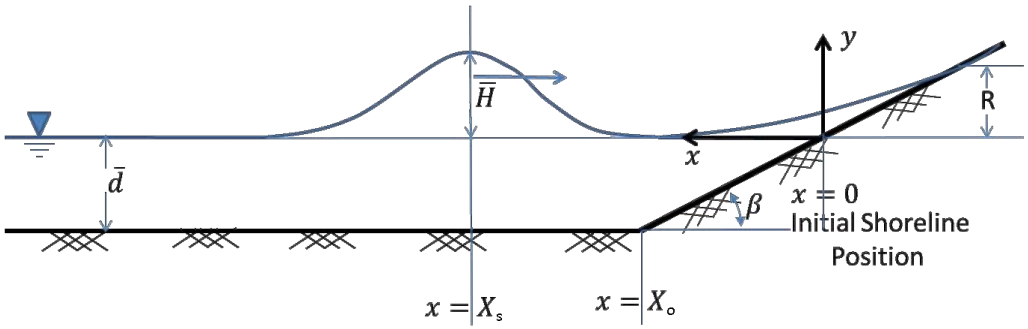
Ignore dispersion locally and momentarily if:

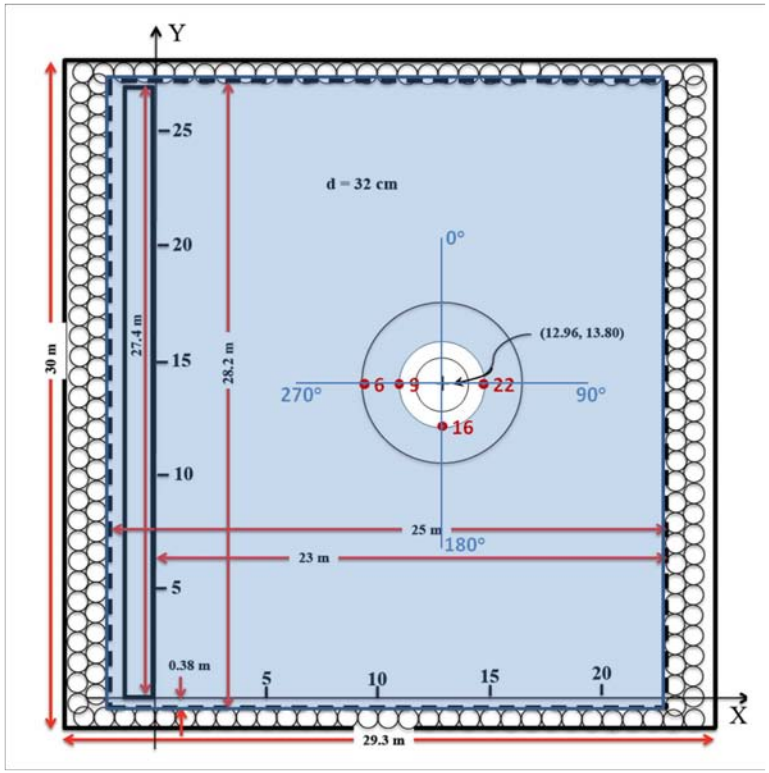
$$\left|\frac{\partial(HU)}{\partial x}\right| > 0.5\sqrt{gH} \quad , \quad \left|\frac{\partial(HV)}{\partial y}\right| > 0.5\sqrt{gH}$$

Previous BOSZ Validation

US National Tsunami Hazard Mitigation Program

Model verification/validation with analytical solutions and laboratory experiments





Previous Model Validation

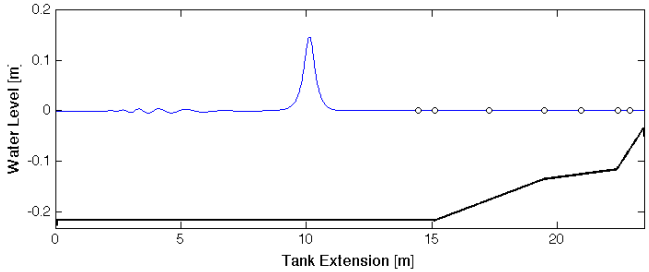
2. Solitary wave reflection

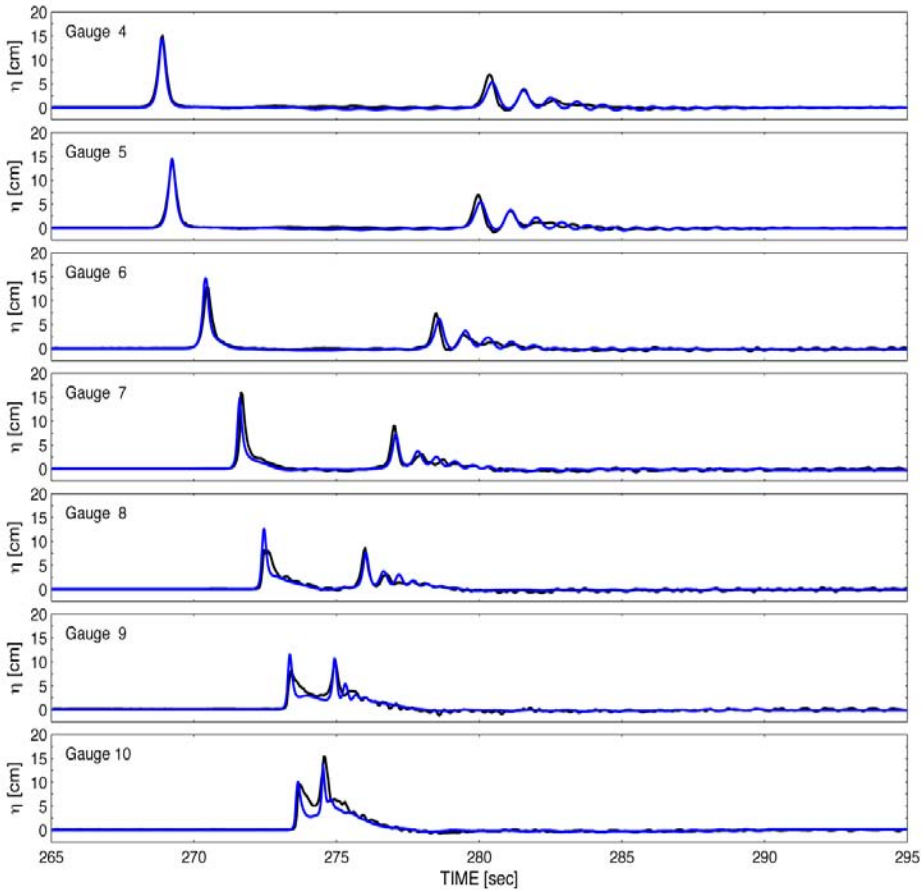
Test C: *A/h* = 0.696

BOSZ
experimental data (Briggs et al., 1995)

BOSZ performance:

- Reasonable results even for conditions beyond model's range of applicability





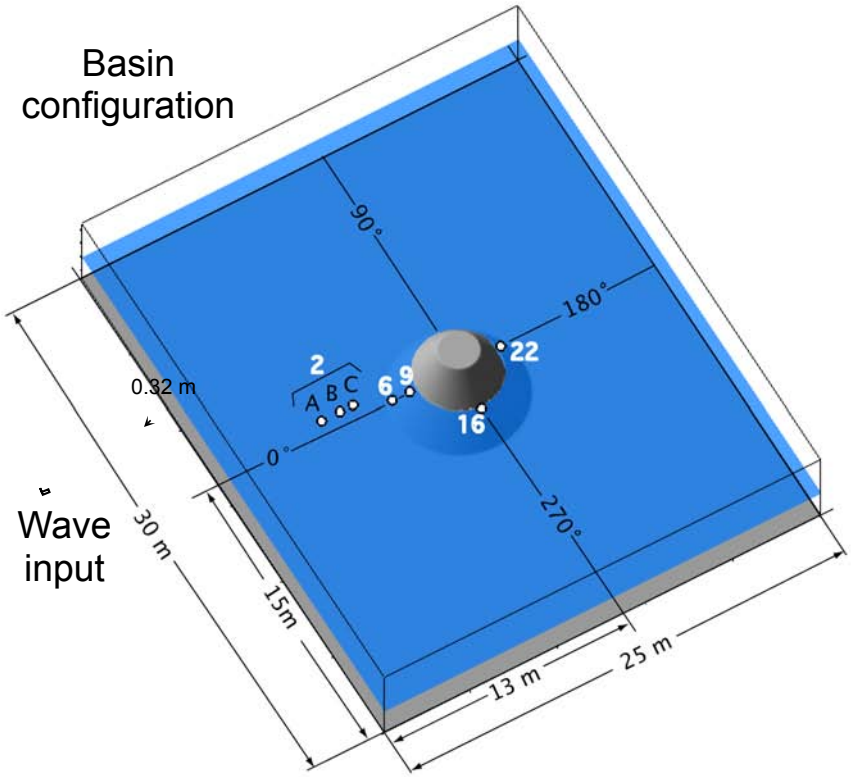
Runup experiment: 27.43 cm
Runup BOSZ: 22.10 cm

81 %

Previous Model Validation

3. Solitary wave around conical island

- BOSZ, no breaking
- BOSZ, with breaking
- experimental data (Briggs et al., 1995)

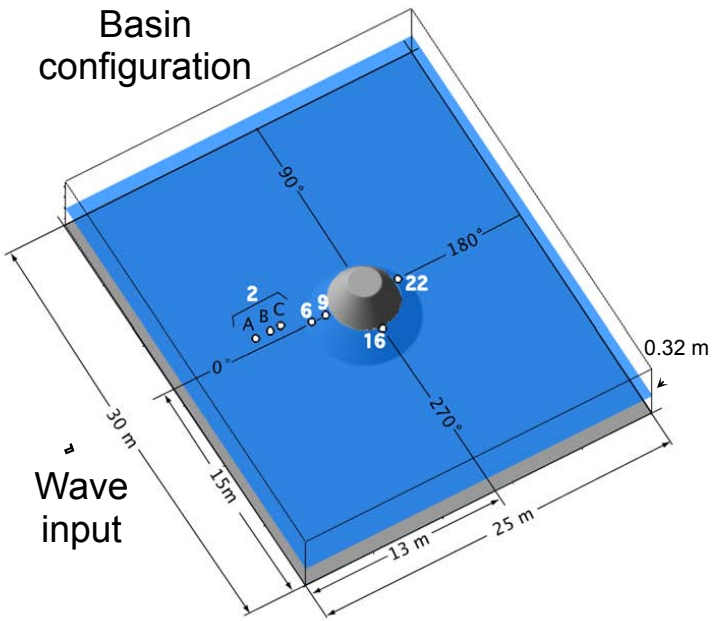


Previous Model Validation

3. Solitary wave around conical island

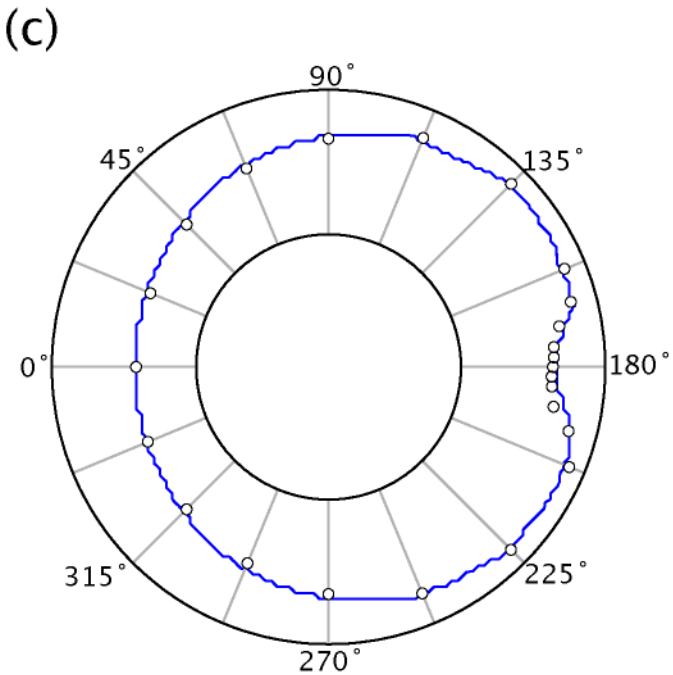
- BOSZ, no breaking
- BOSZ, with breaking
- experimental data (Briggs et al., 1995)

Basin configuration



Wave input

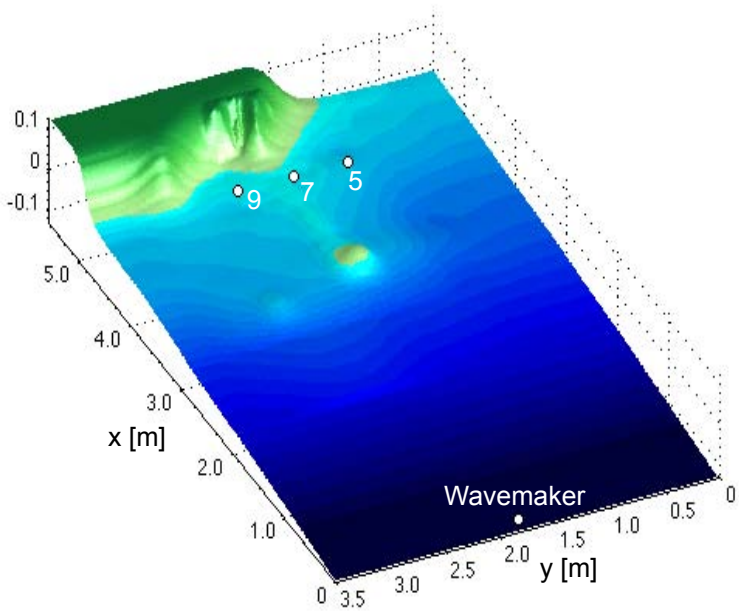
Inundation

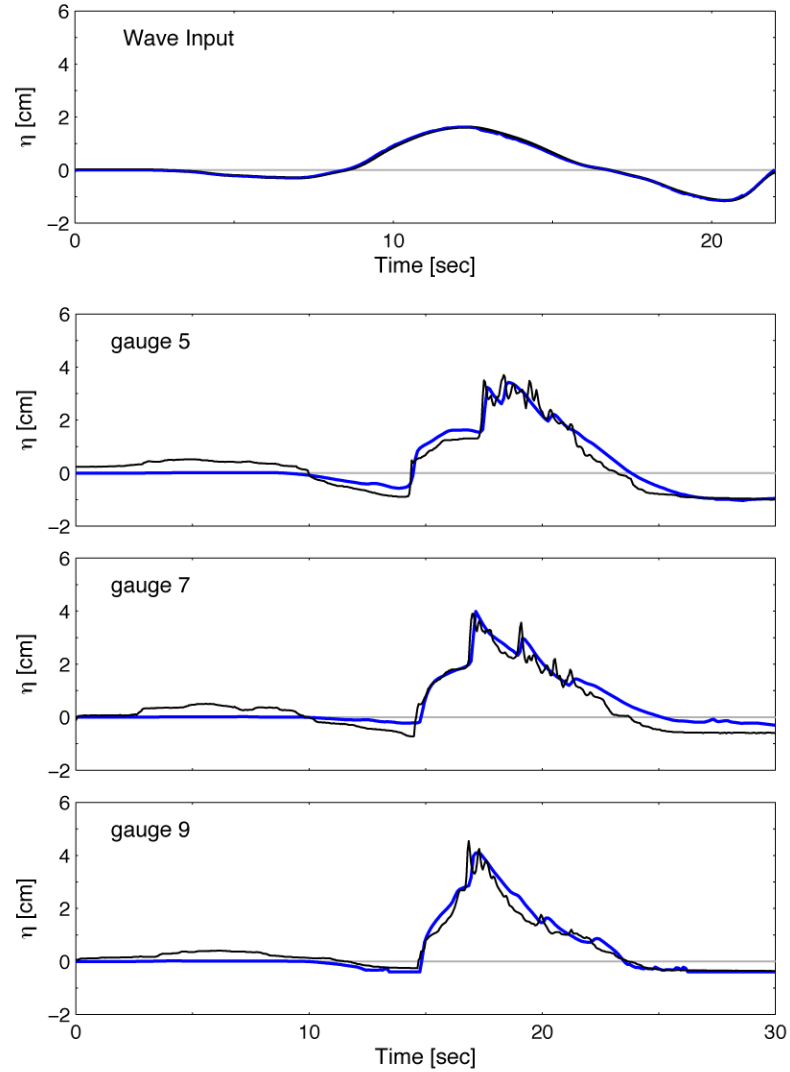


Previous Model Validation

4. N-wave at Okushiri Island, Monai

BOSZ
experimental data
(Matsuyama & Tanaka 2001)





Previous Model Validation

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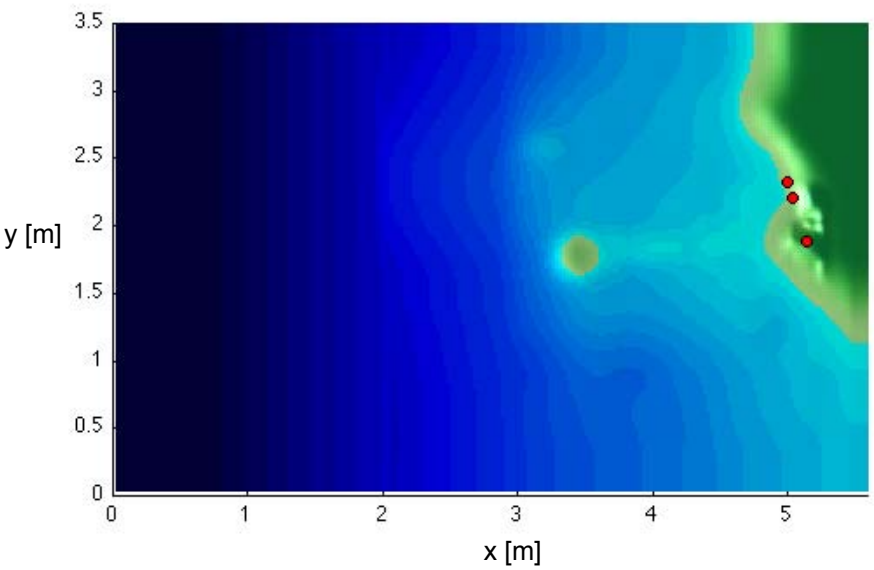
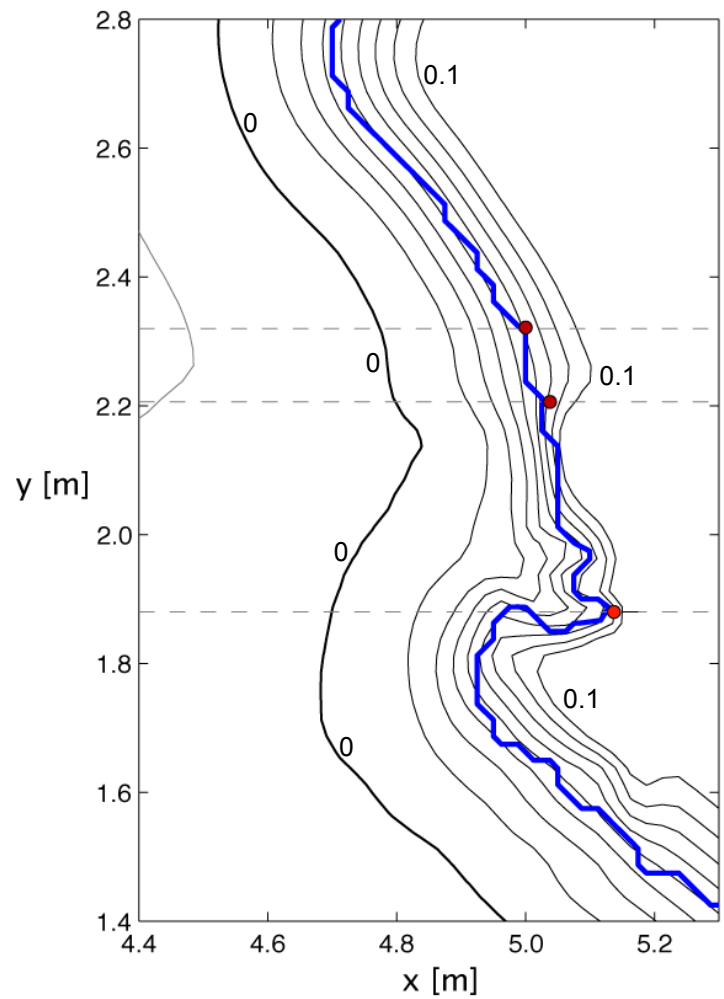
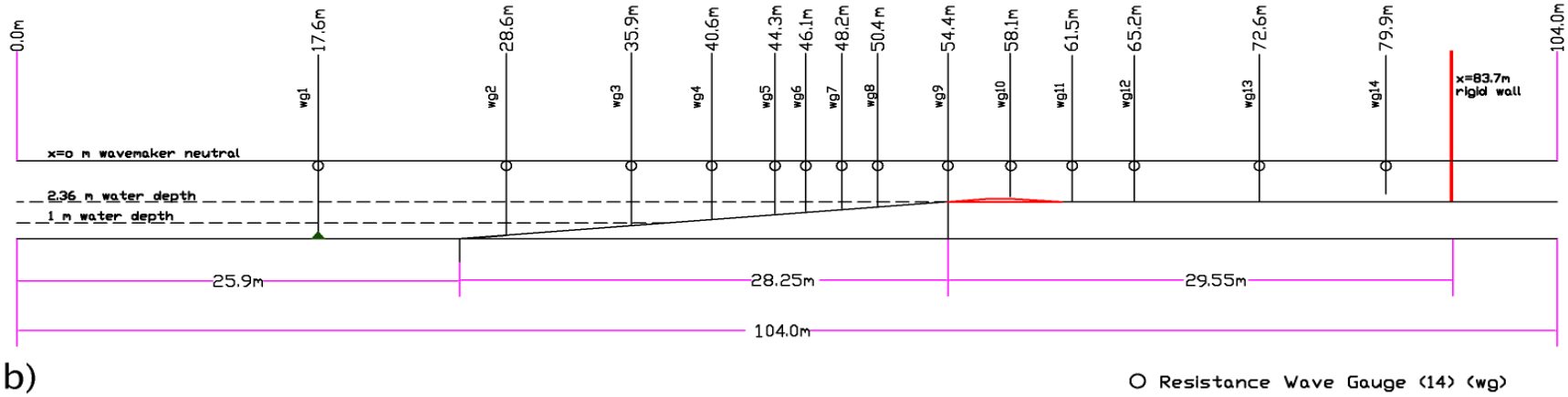


Inundation Line



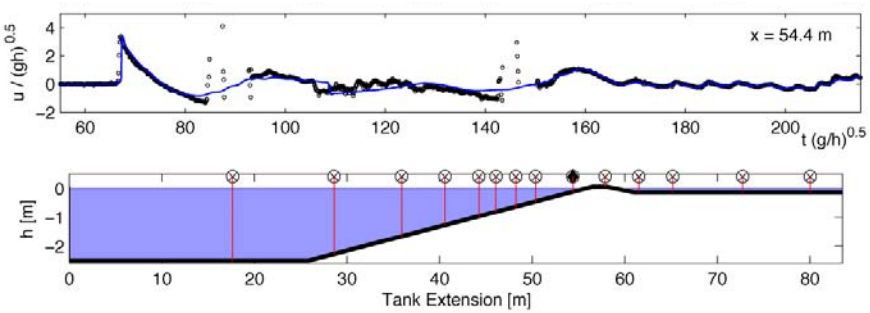
Additional Benchmark Tests

Large Wave Flume,
Oregon State University



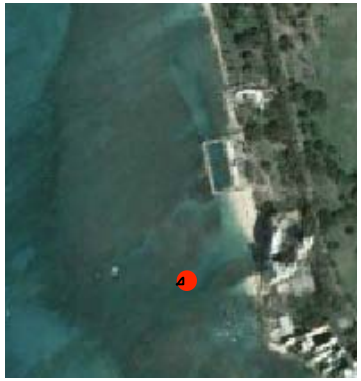
b)

O Resistance Wave Gauge (14) (wg)

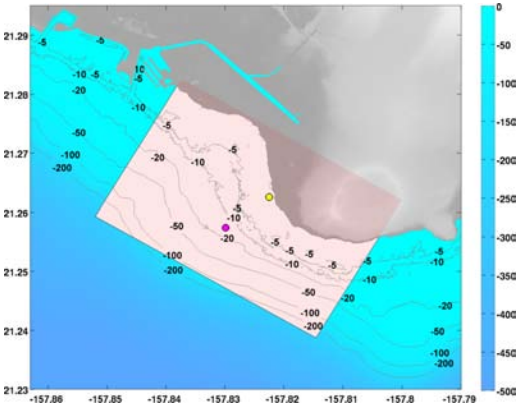


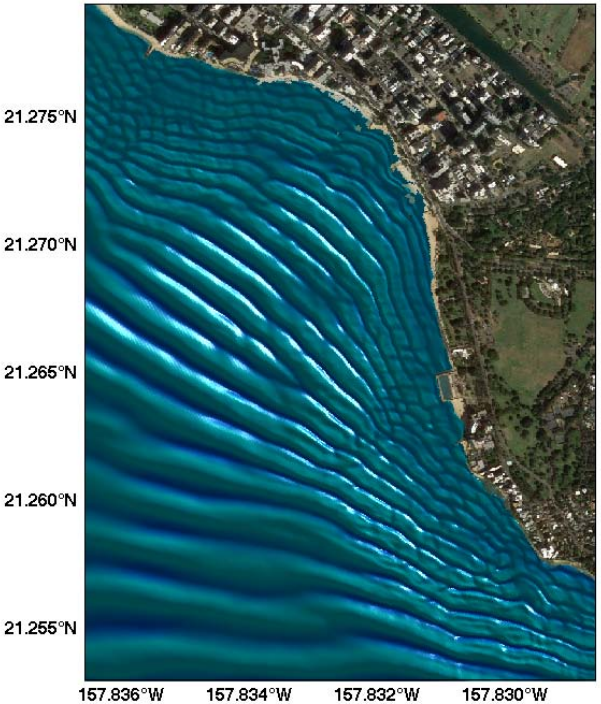
BOSZ - validation, *real reef*

Southshore Oahu, HI

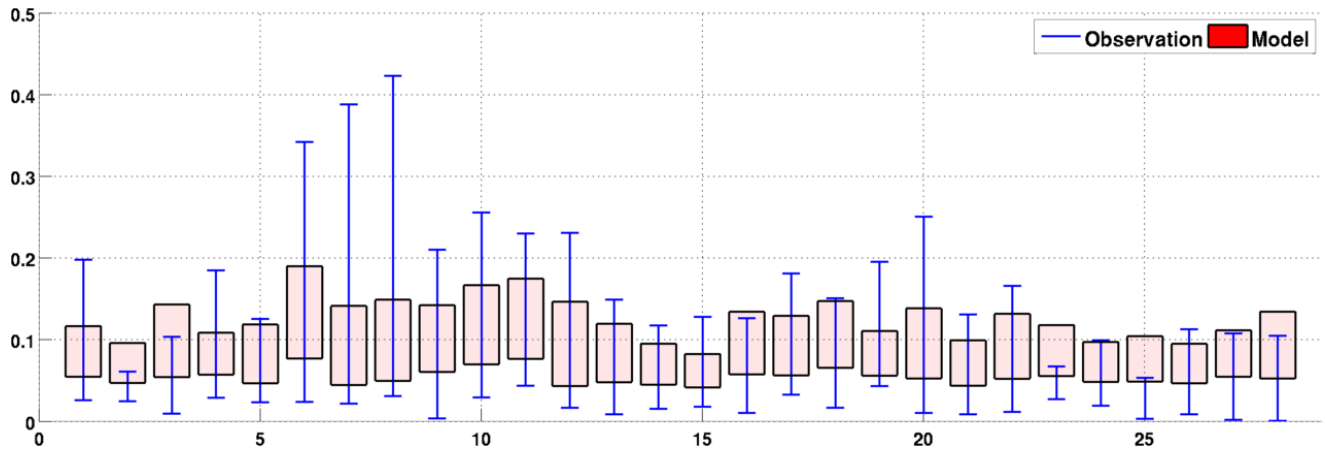


Aquadopp ADCP

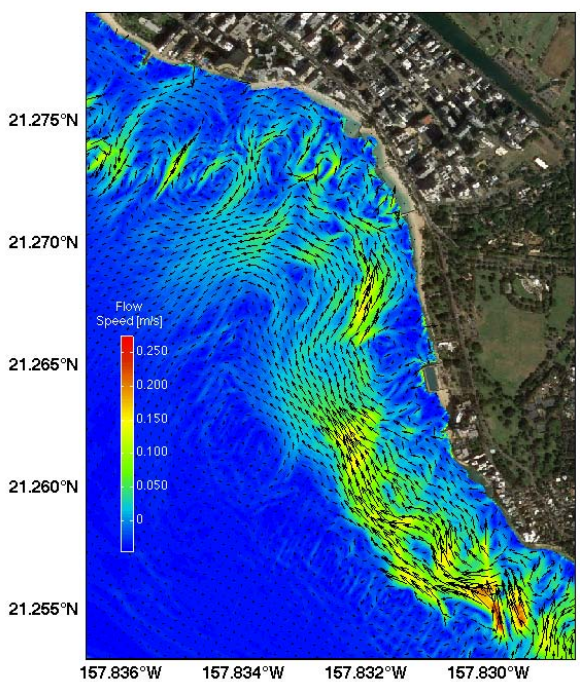




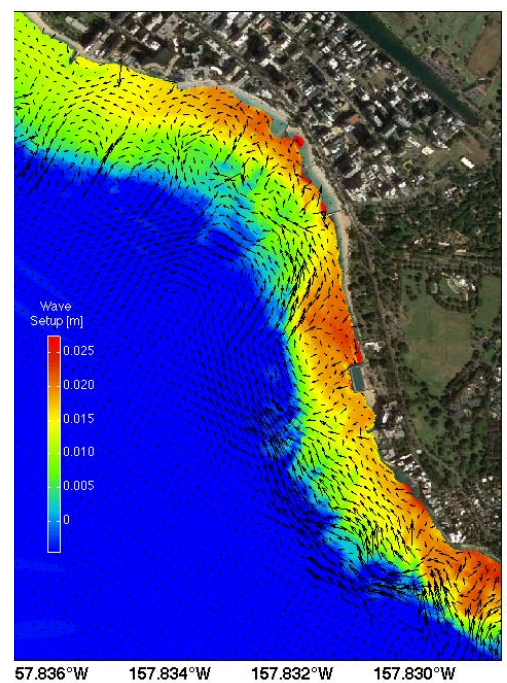
BOSZ – flow velocity, Waikiki



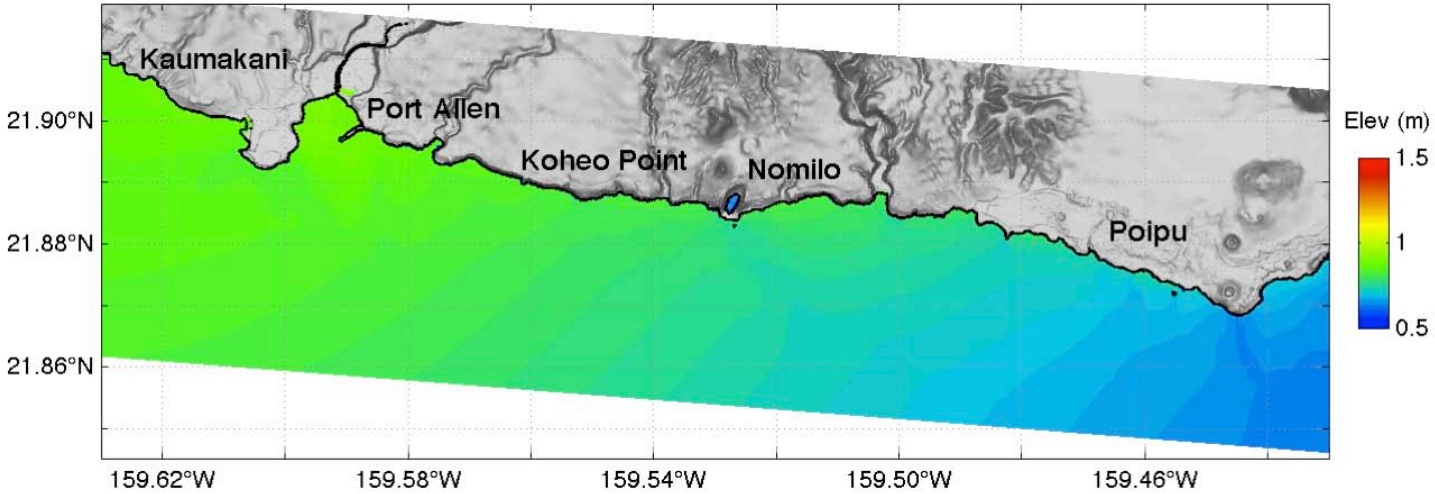
Mean flow velocity

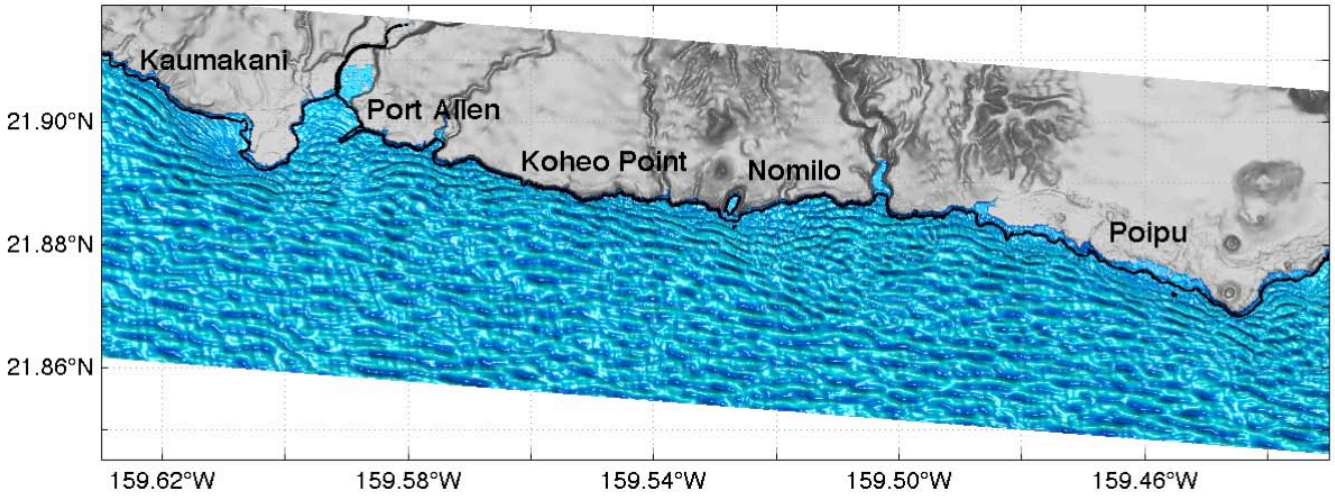


Wave Setup

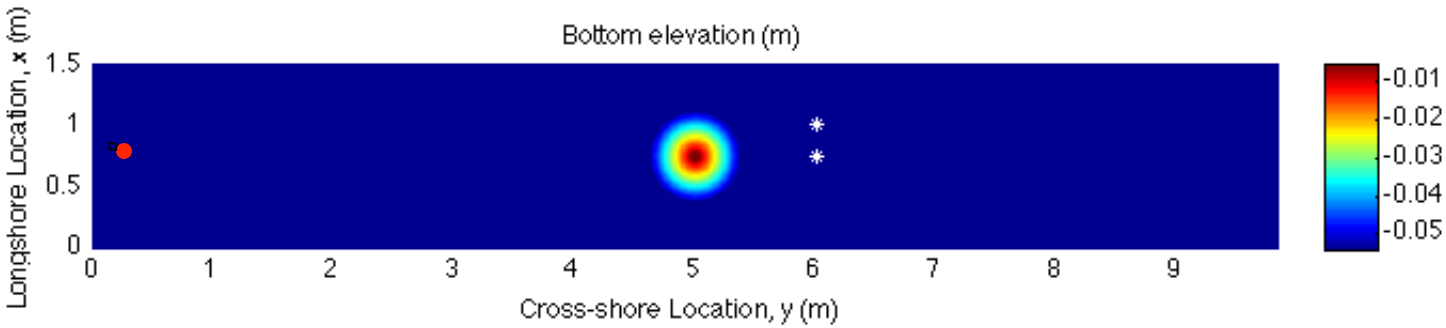


Nearshore Waves on top of storm surge





BM1, *flow over cone*

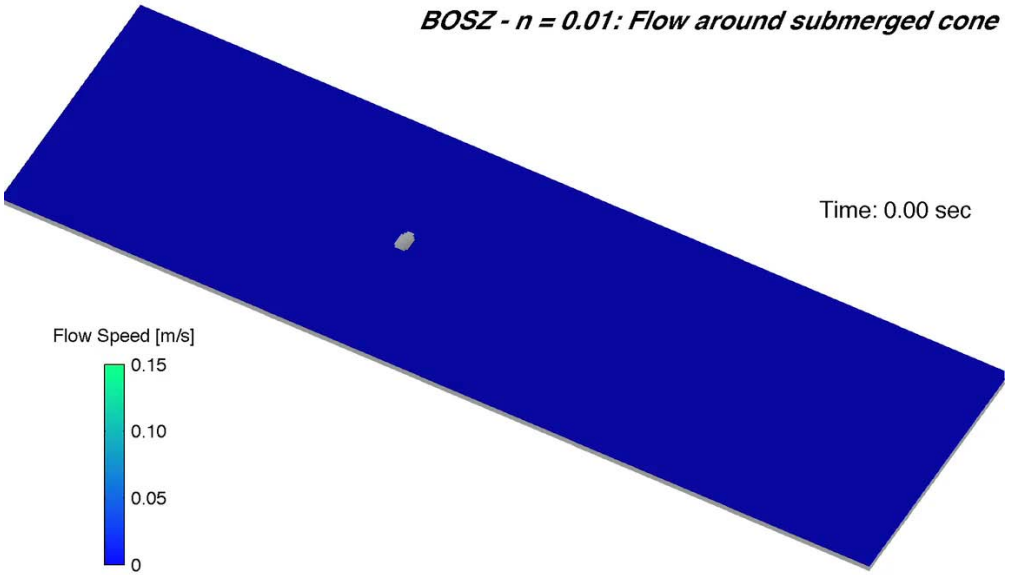


Flume:
Water depth: **5.4** cm
Flow speed : **11.5** cm/s
Courant : **0.158**

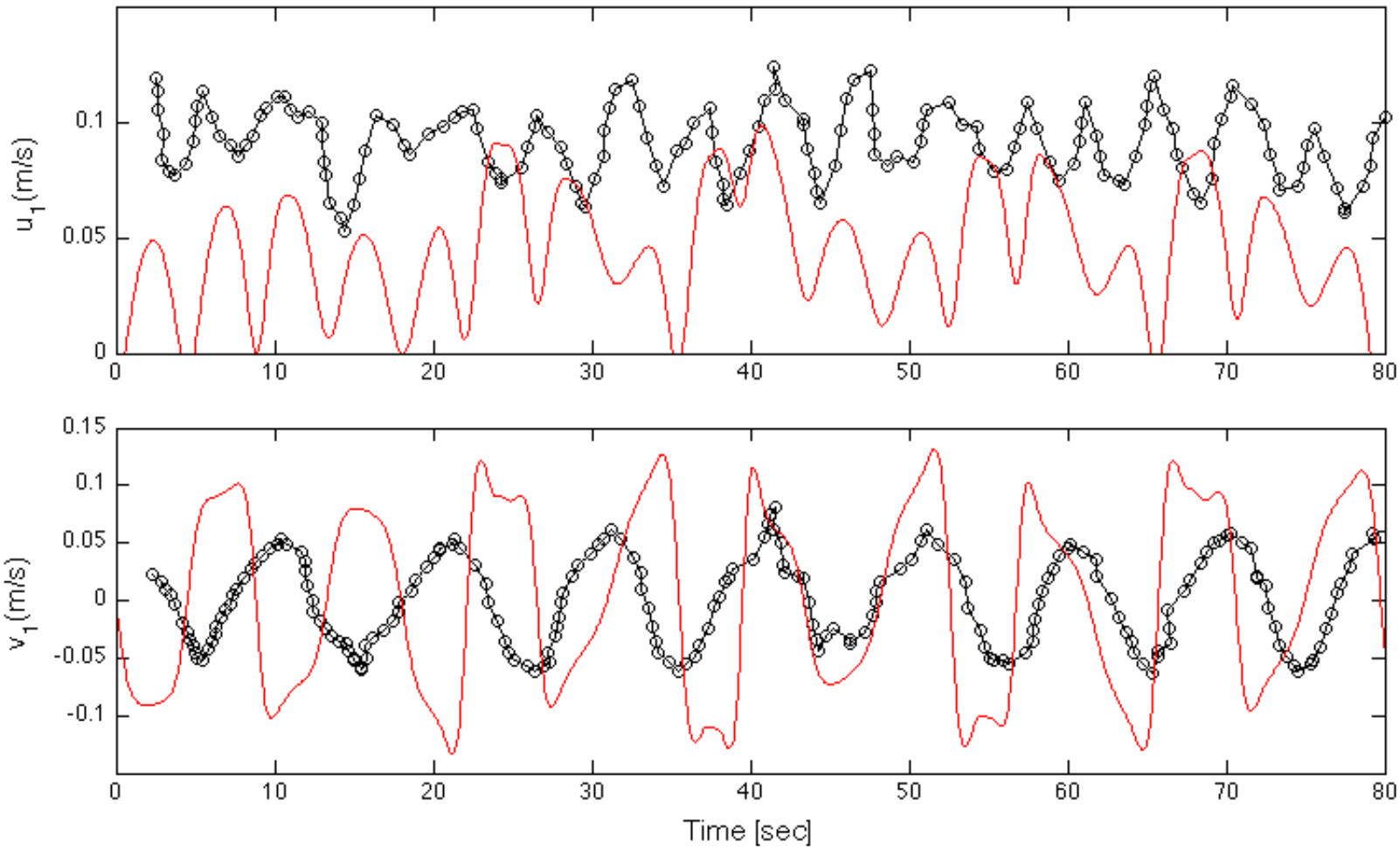
BOSZ settings:
Δx : 1-3 cm
n : 0.01-0.025 $s/m^{1/3}$

Boundary conditions:

- Discharge at left boundary by adding mass
- Iterative adjustment of flow speed (control point)
- Subtraction of extra flow depth from bathymetry

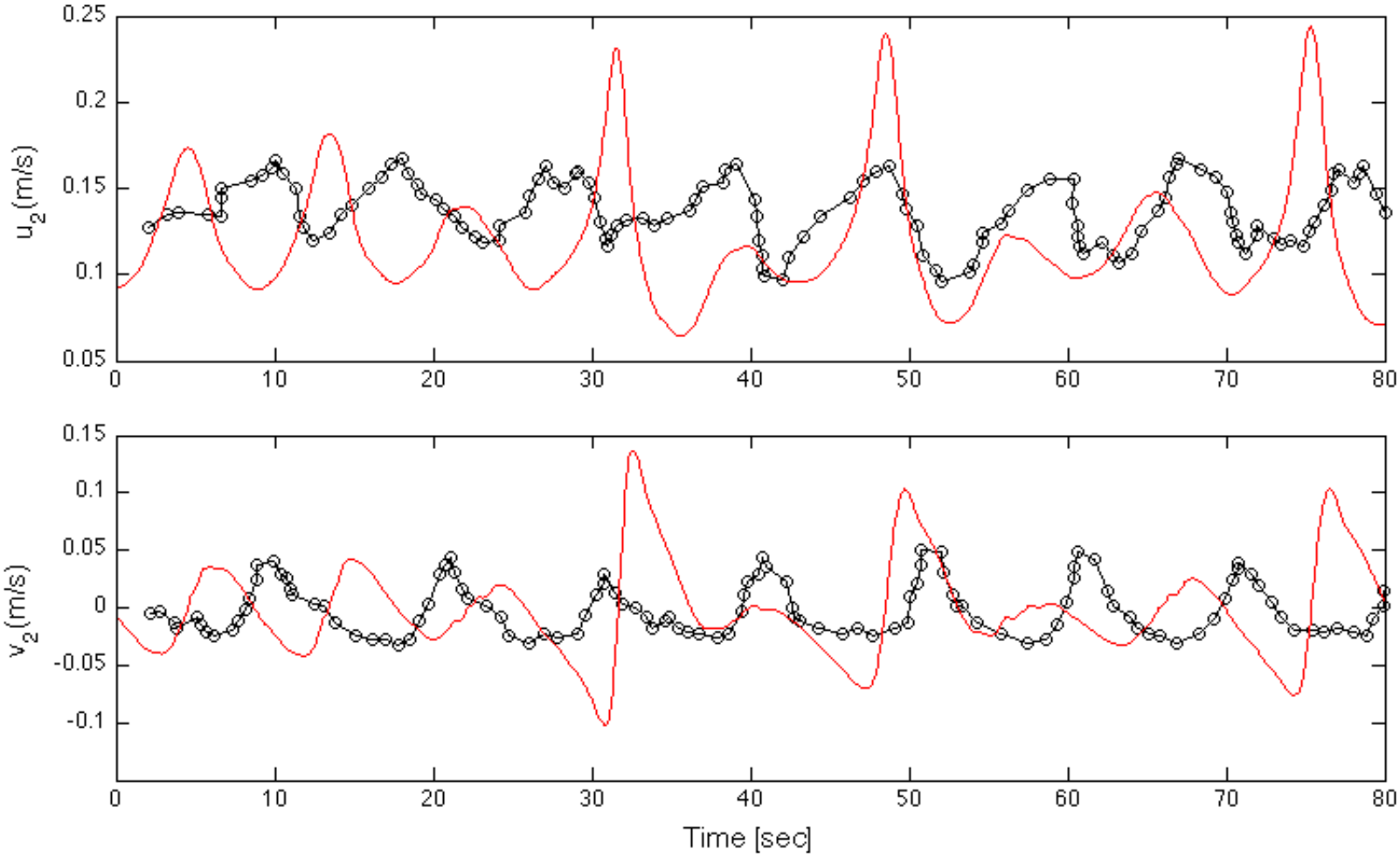

BOSZ - n = 0.01: Flow around submerged cone
Time: 0.00 sec
Flow Speed [m/s]
0.15
0.10
0.05
0
BOSZ - n = 0.01
Time: 0.00 se
Vorticity [Hz]
3.0
0
-3.0

Comparison at Gauge 1

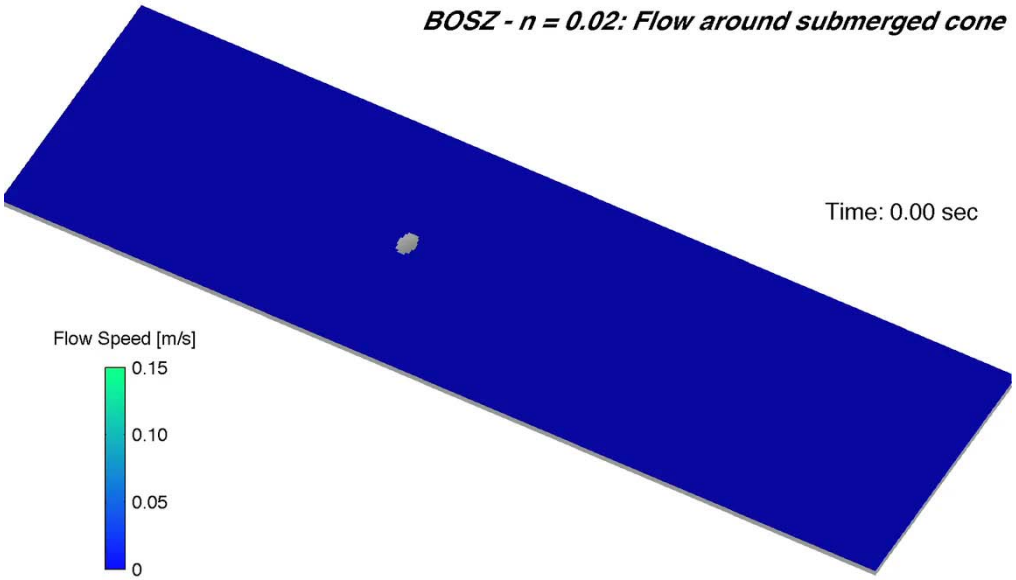


n= 0.01 s/m$^{1/3}$

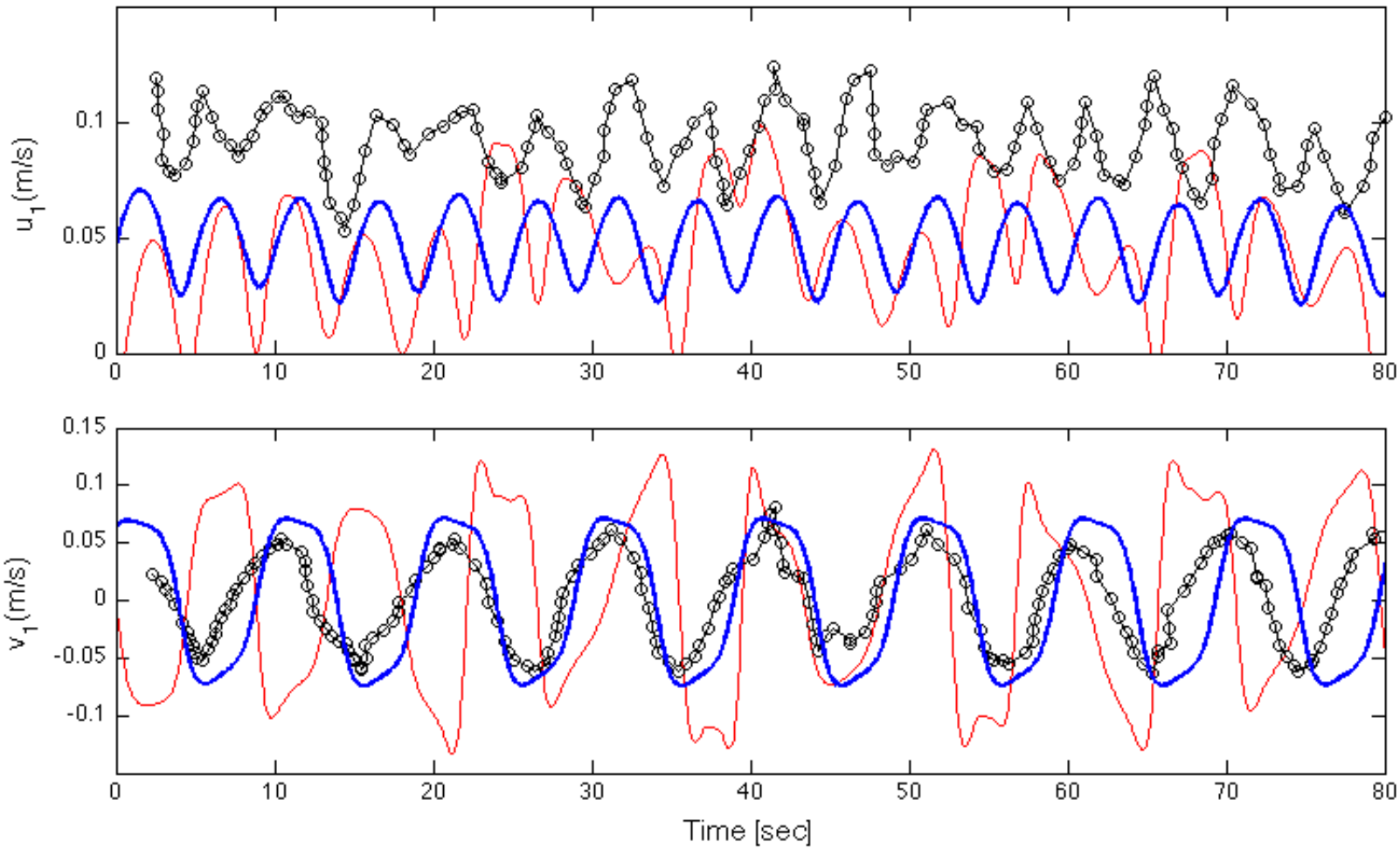
Comparison at Gauge 2



n= 0.01 s/m$^{1/3}$


BOSZ - n = 0.02: Flow around submerged cone
Time: 0.00 sec
Flow Speed [m/s]
0.15
0.10
0.05
0
BOSZ - n = 0.02
Time: 0.00 se
Vorticity [Hz]
3.0
0
-3.0

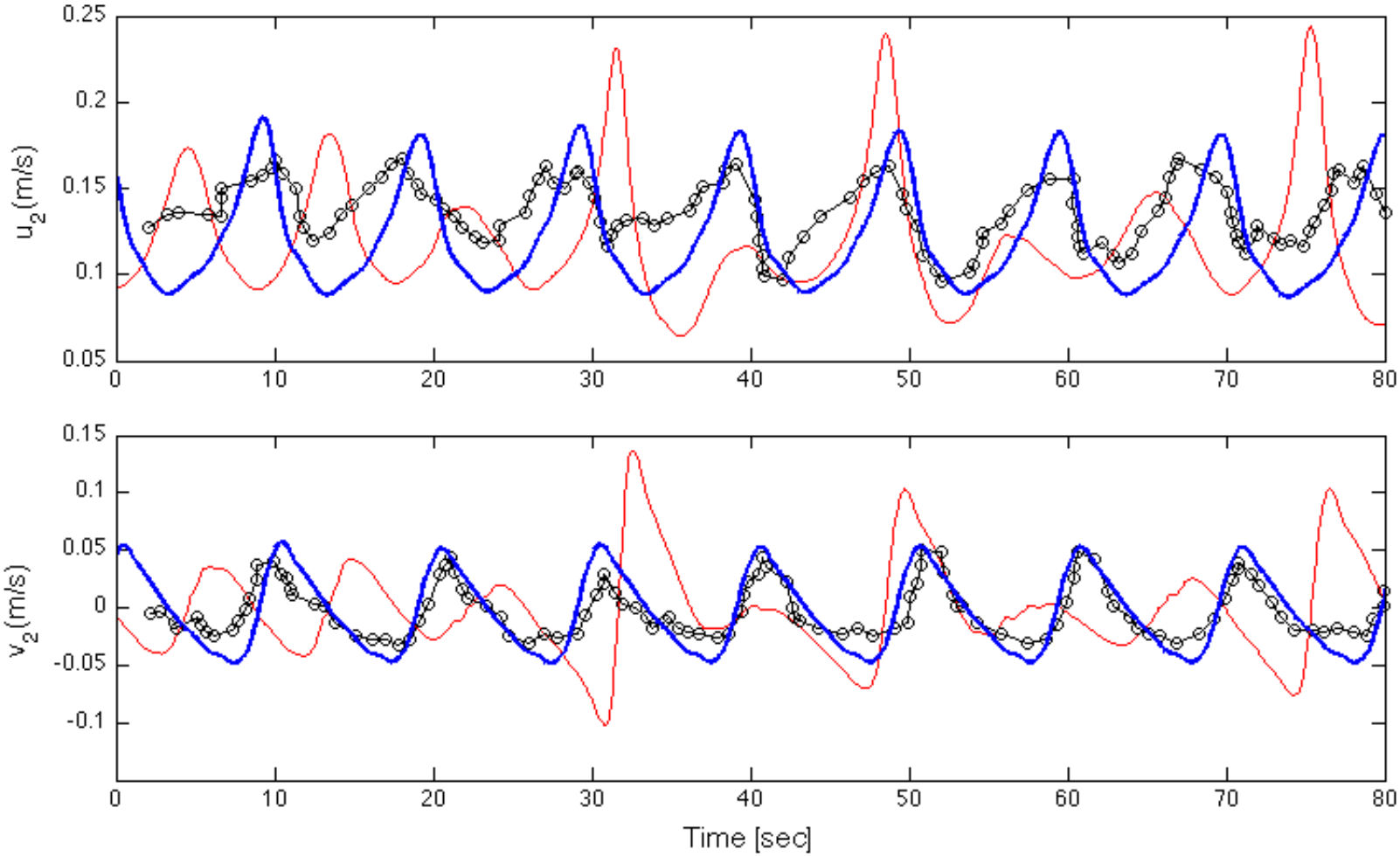
Comparison at Gauge 1



n= 0.01 s/m$^{1/3}$

n= 0.02 s/m$^{1/3}$

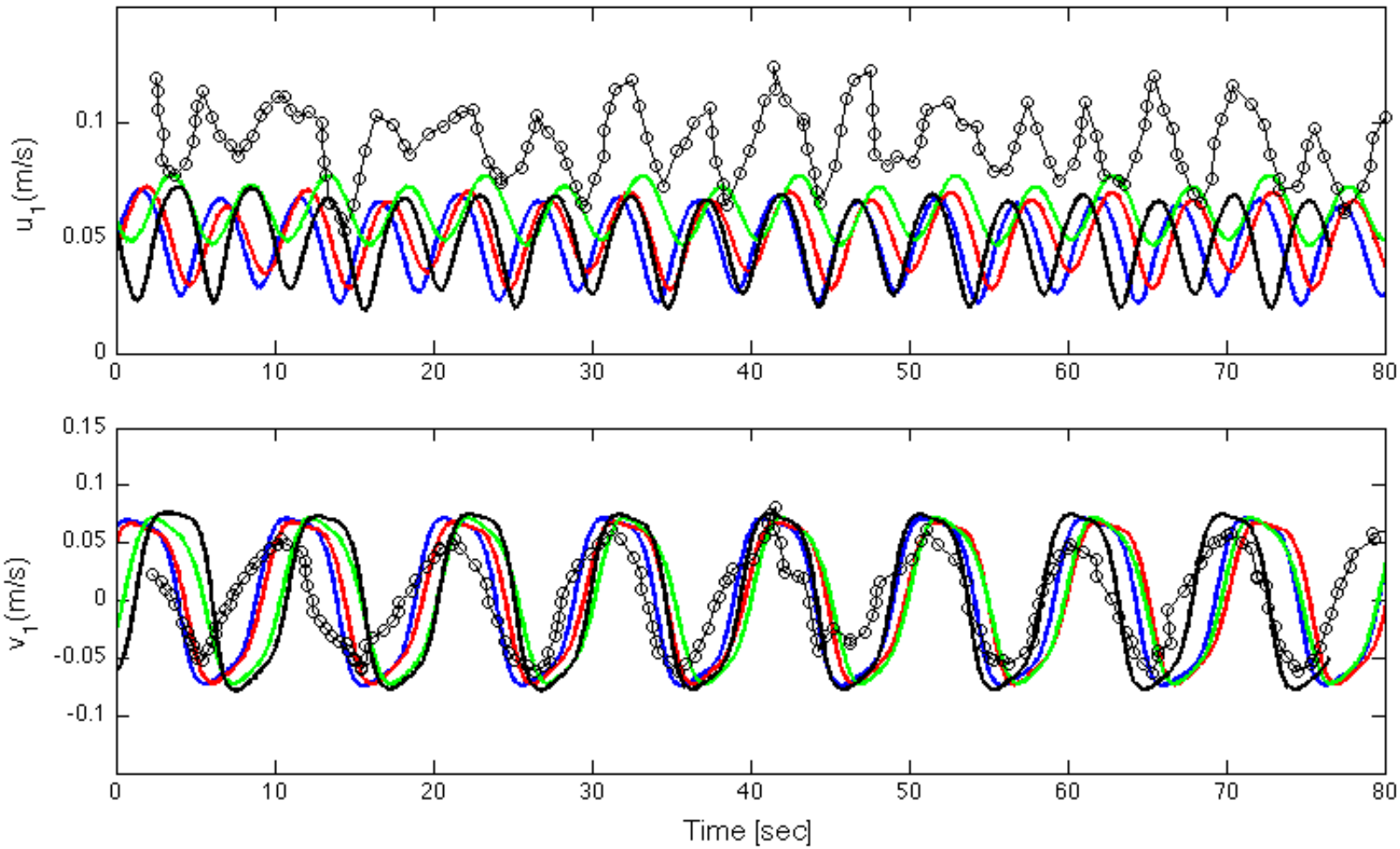
Comparison at Gauge 2



n= 0.01 s/m$^{1/3}$

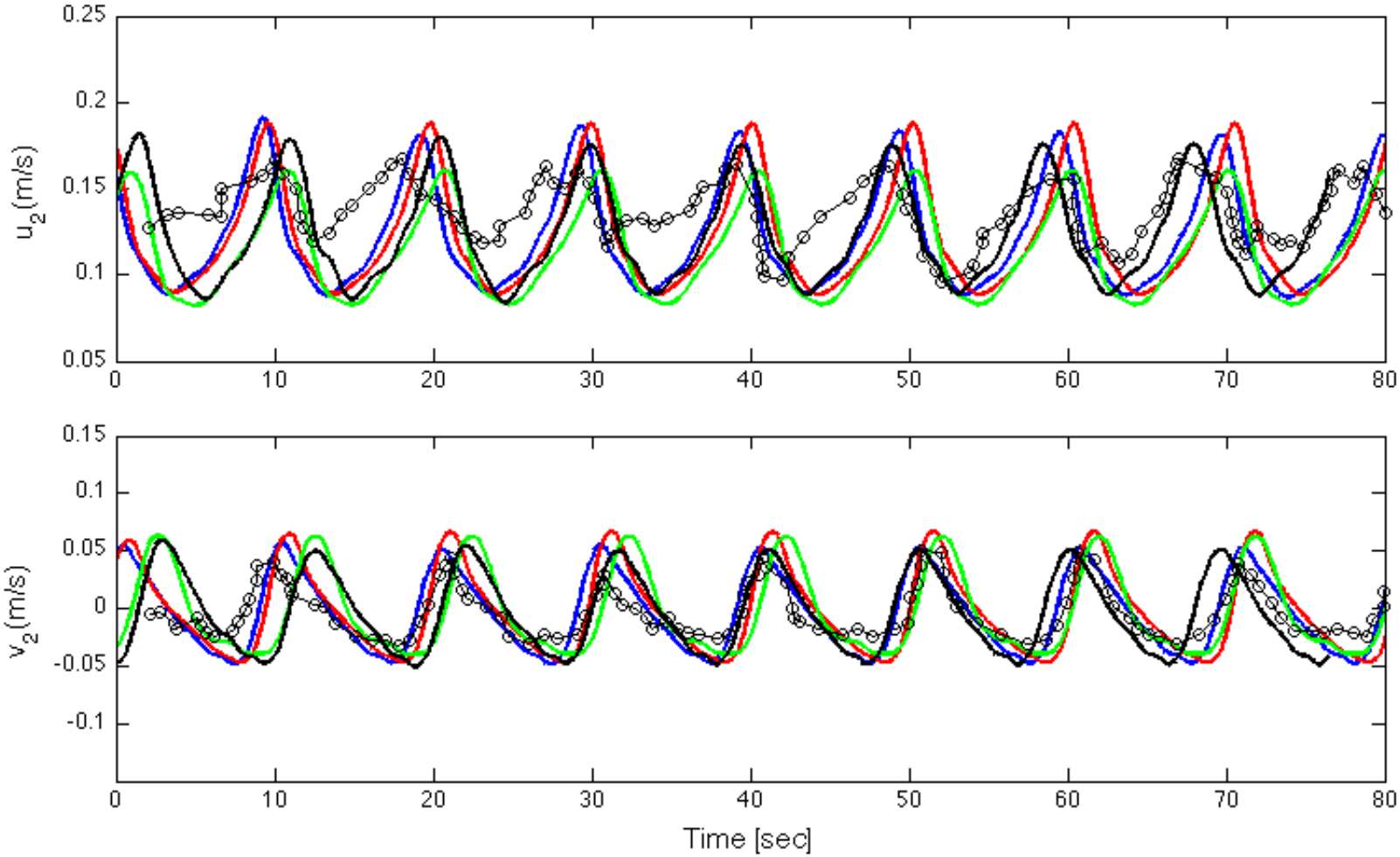
n= 0.02 s/m$^{1/3}$

Grid size dependency



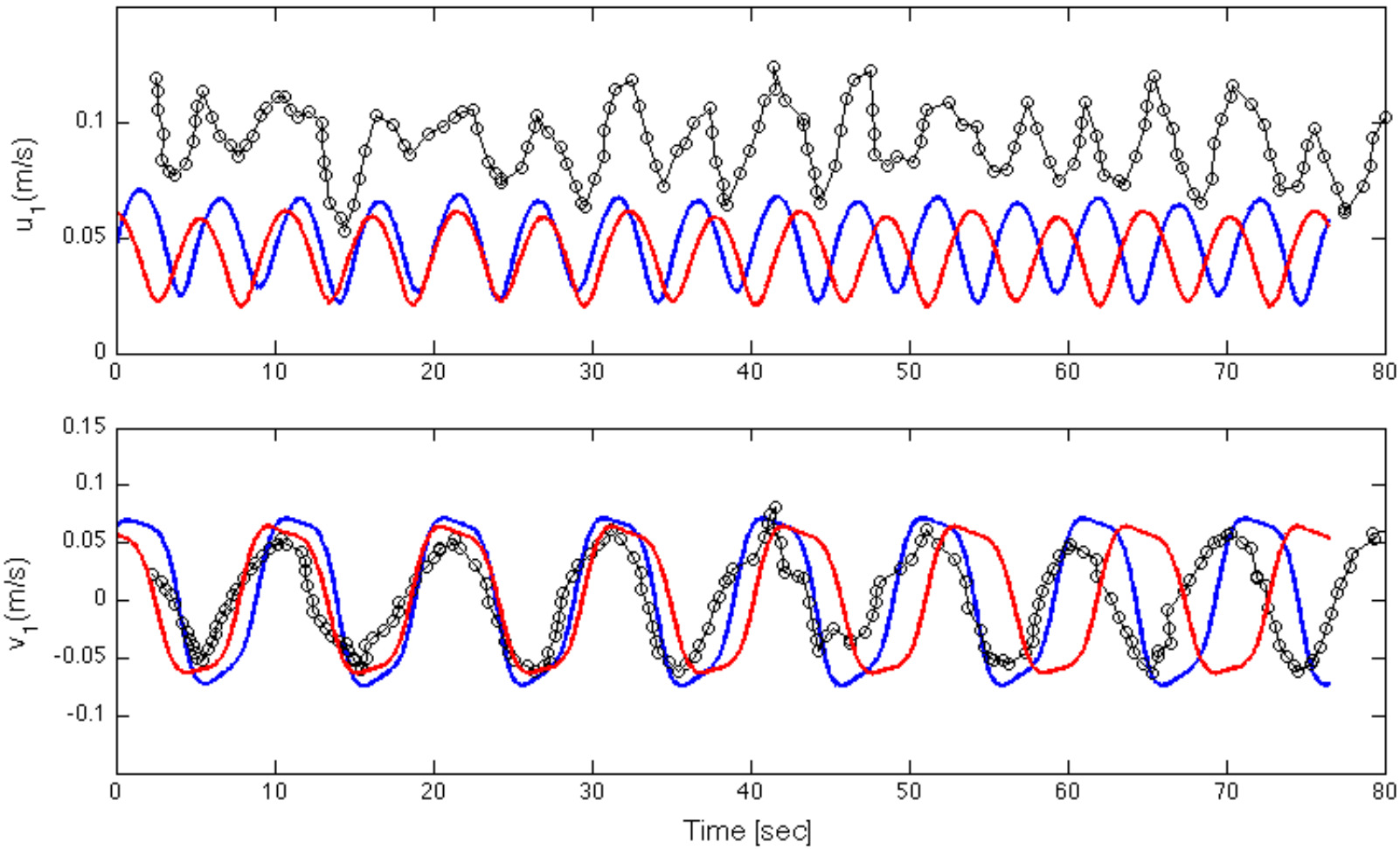
dx = 0.010 m *dx* = 0.015 m *dx* = 0.020 m *dx* = 0.030 m

Grid size dependency



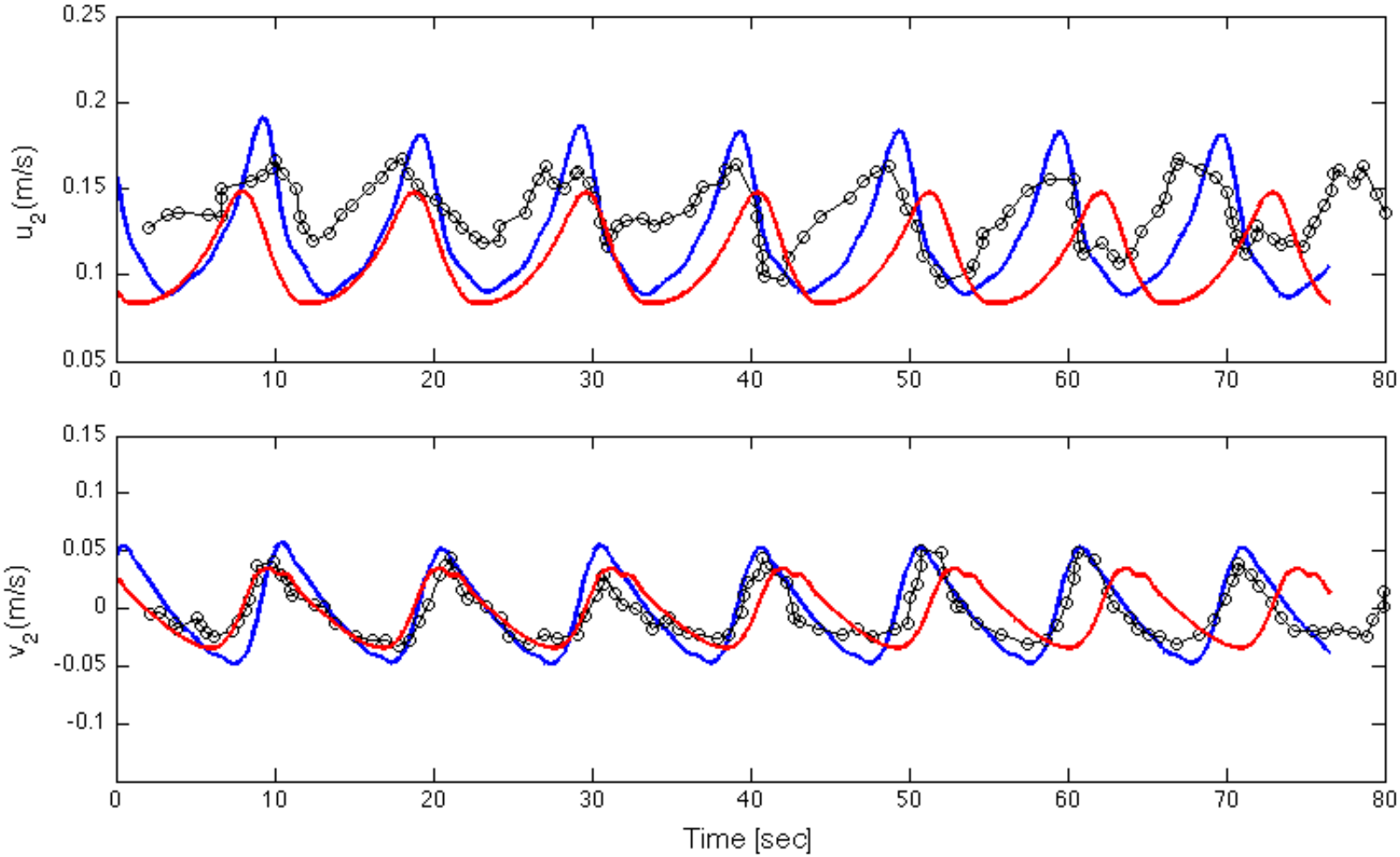
dx = 0.010 m *dx* = 0.015 m *dx* = 0.020 m *dx* = 0.030 m

dispersive vs. hydrostatic



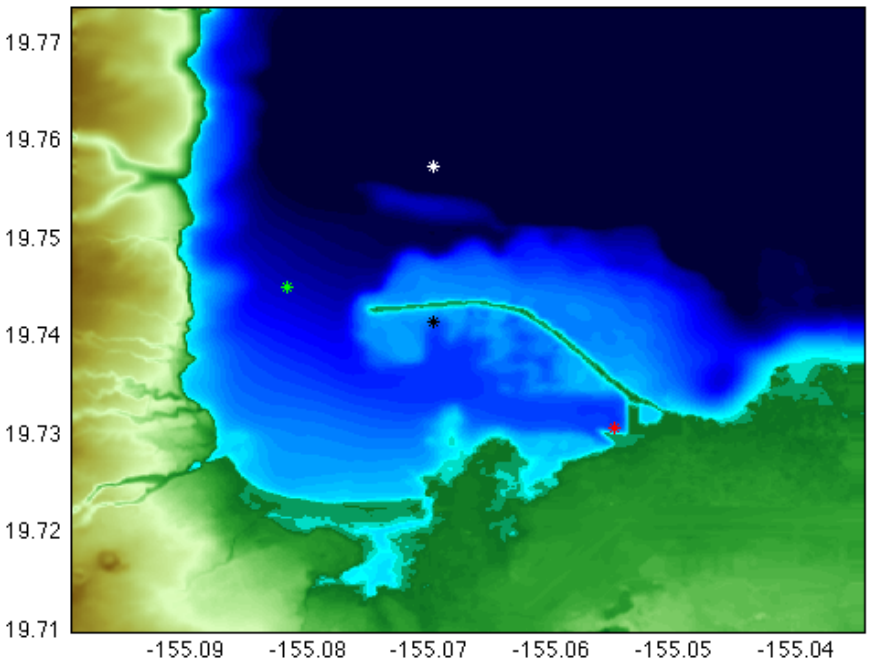
dispersive *hydrostatic*

dispersive vs. hydrostatic



dispersive *hydrostatic*

BM2, *Tohoku tsunami in Hilo Bay*



Δy

Δx

BOSZ settings:

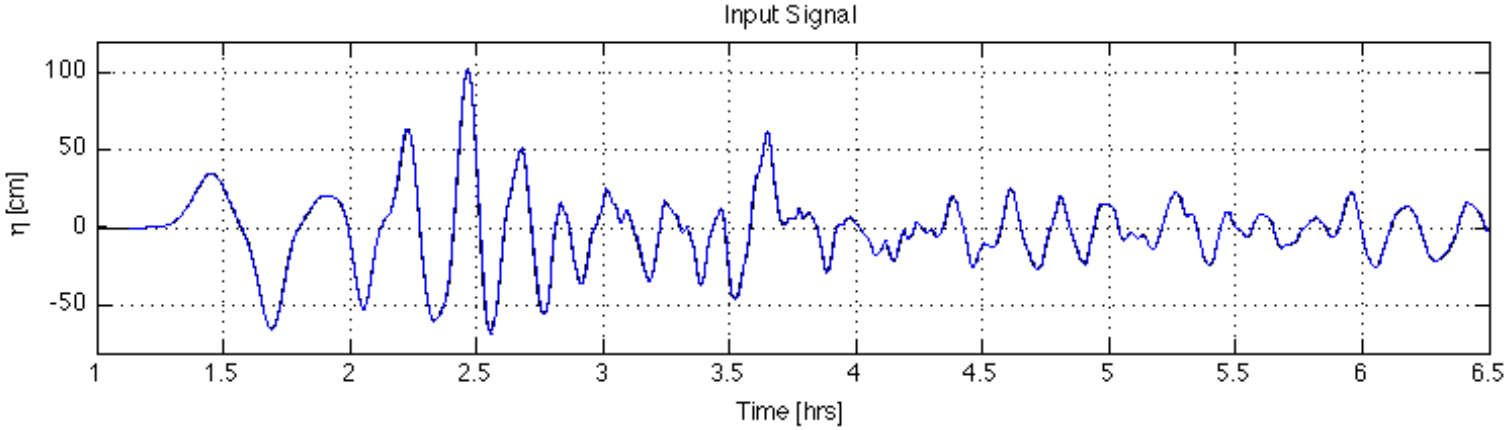
$\Delta x, \Delta y$: 18.7 m / 20.6 m
9.3 m / 10.3 m
4.7 m / 5.1 m

n : 0.025 $s/m^{1/3}$

Boundary conditions:

- Change in continuity equation according to tsunami time series
- Input only along North boundary
- Radiation boundary conditions at North and East side

Initial test with input wave signal over flat bathymetry



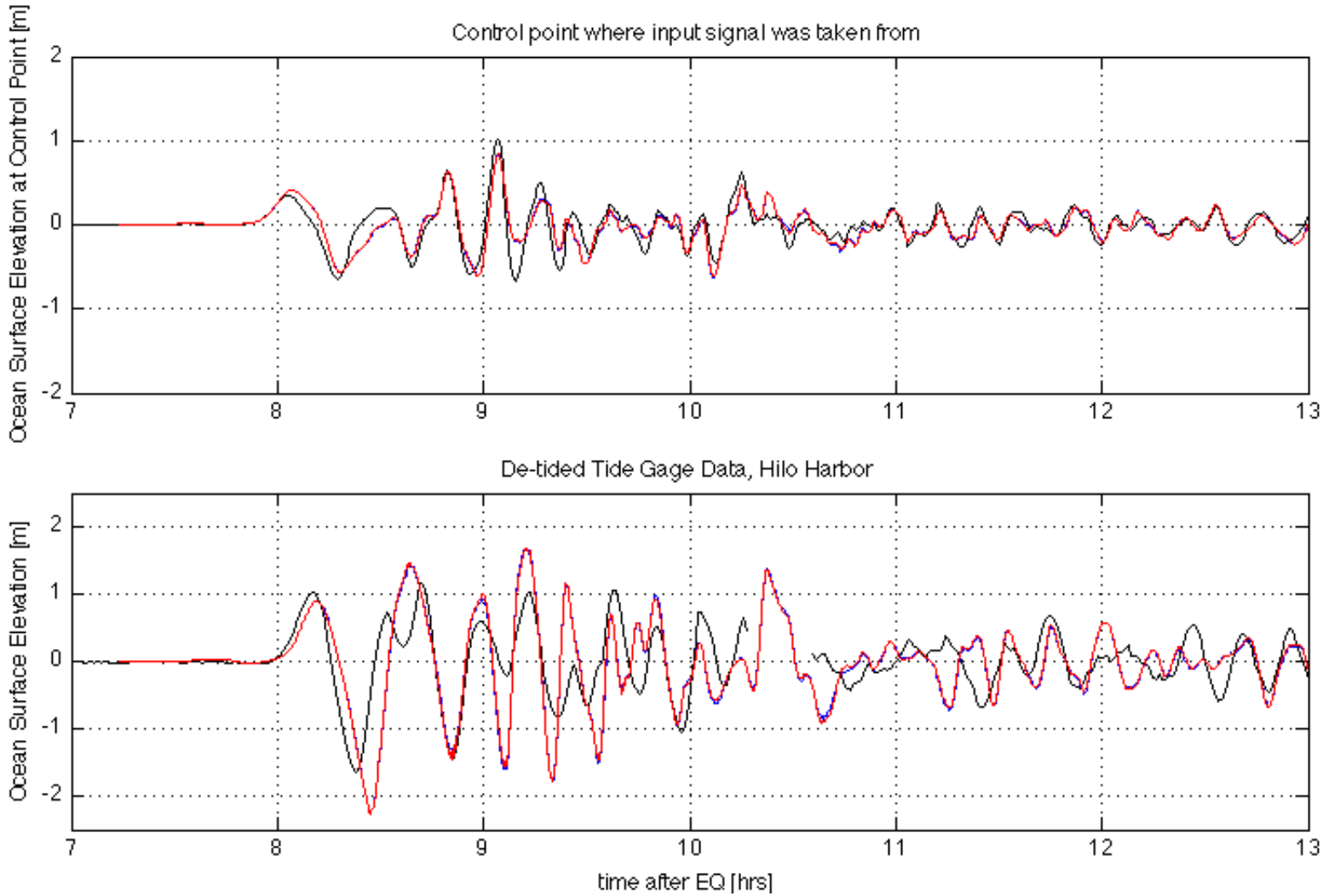
Data

BOSZ

- uniform bathymetry of 30 m
- no friction
- open radiation boundary conditions

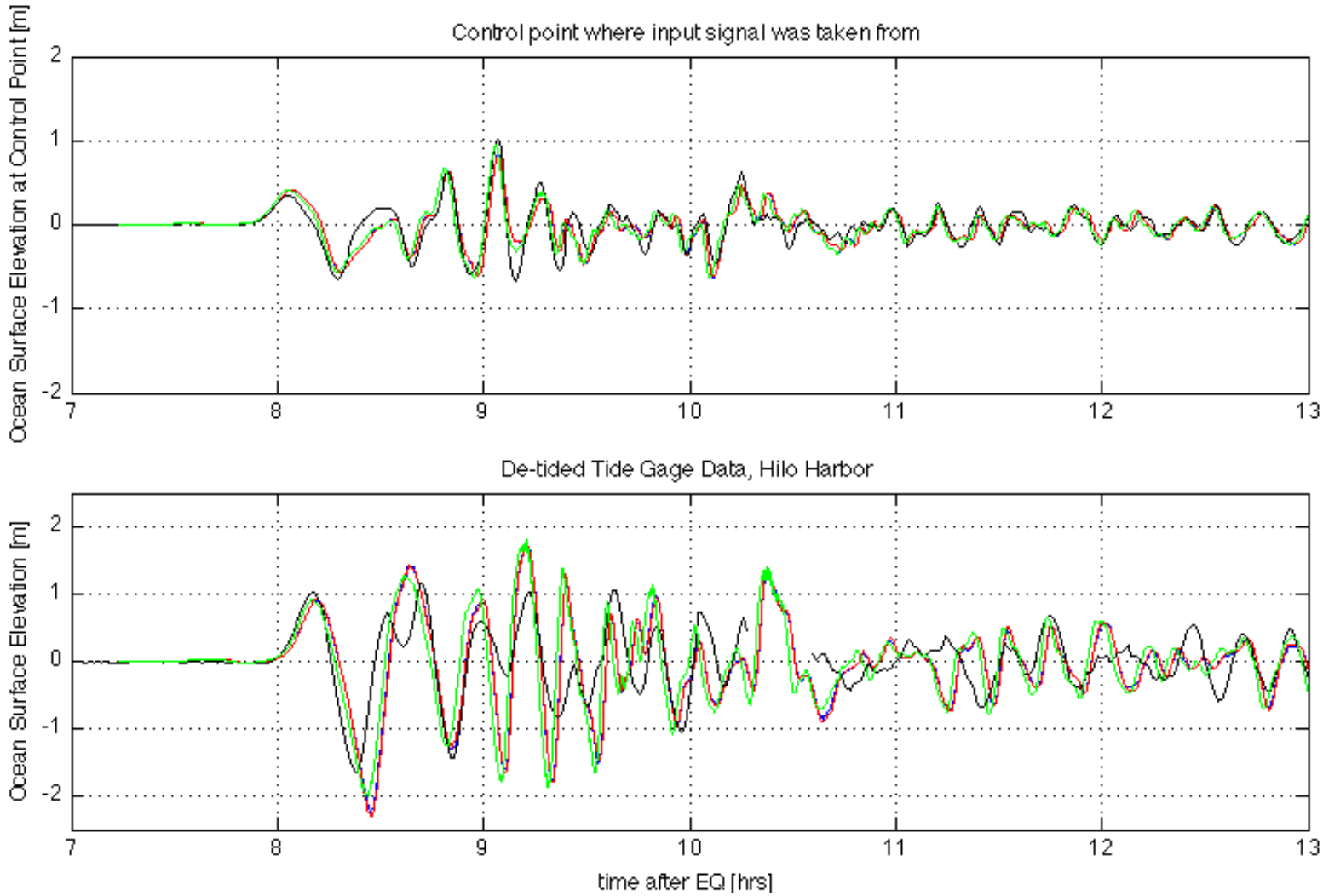
BOSZ - BM2, Hilo Bay

Dispersive vs. hydrostatic



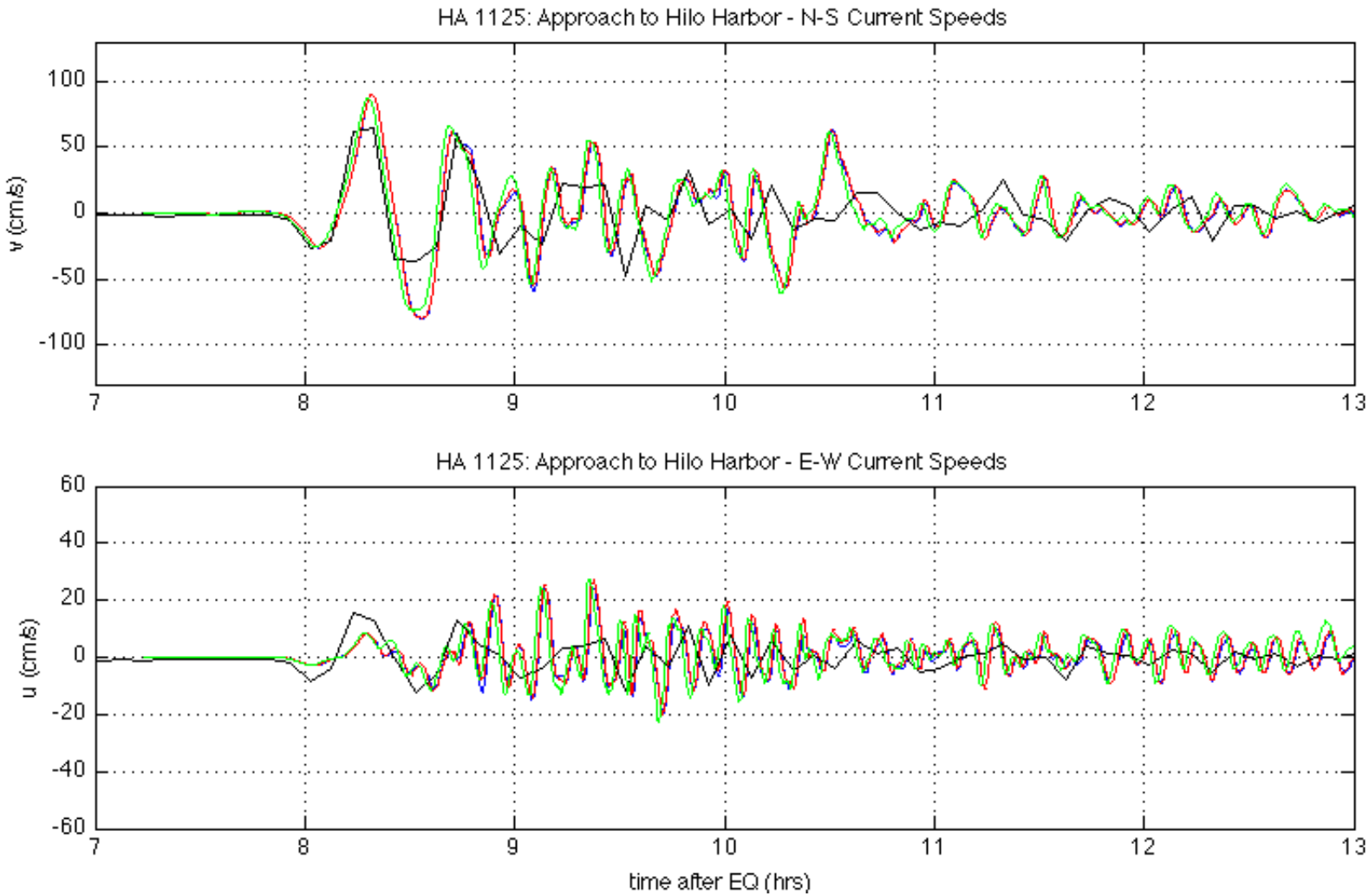
Dispersive hydrostatic

Free surface comparison



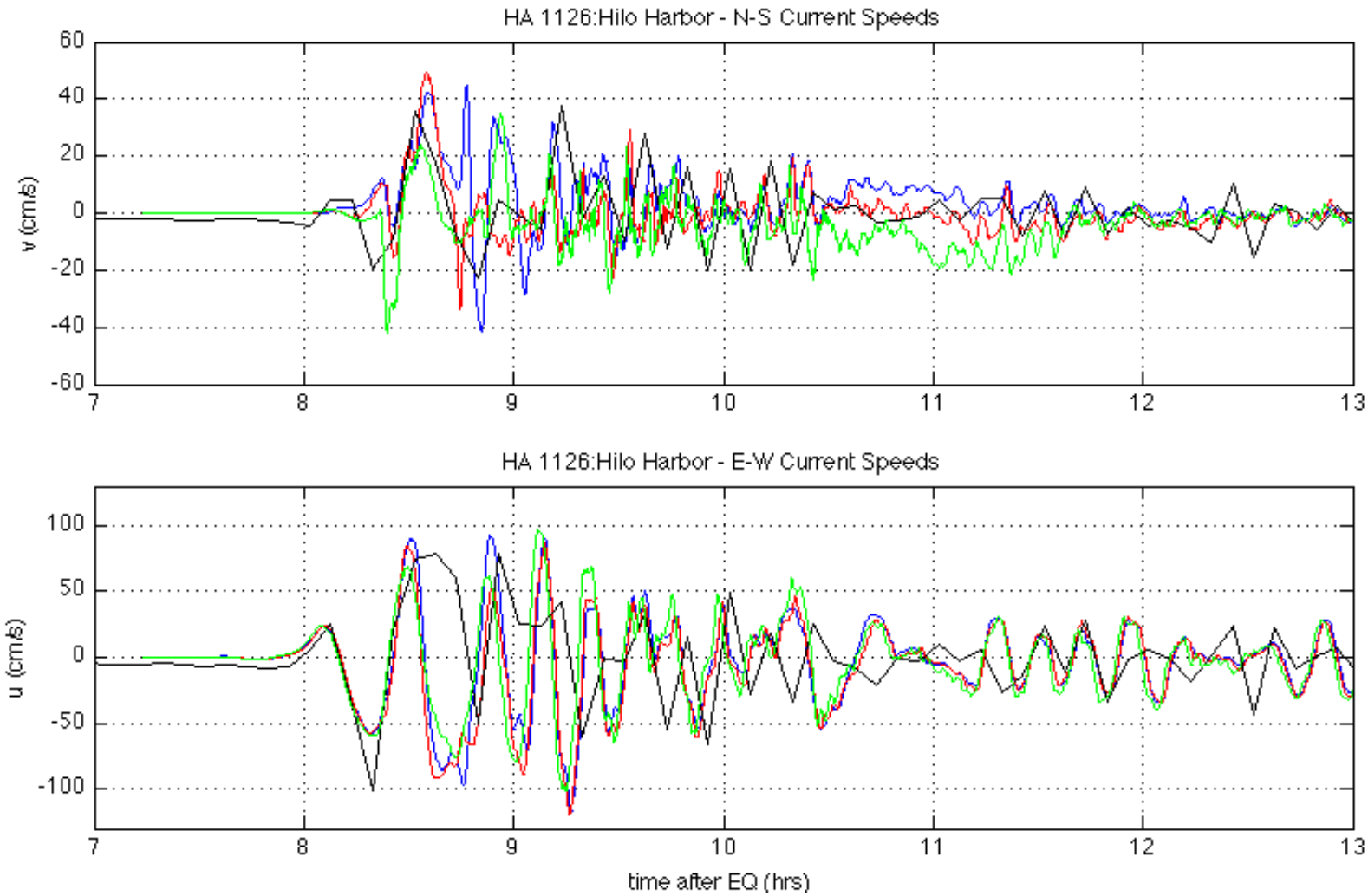
$\Delta x = 20\ m$ $\Delta x = 10\ m$ $\Delta x = 5\ m$

HA1125 - velocity



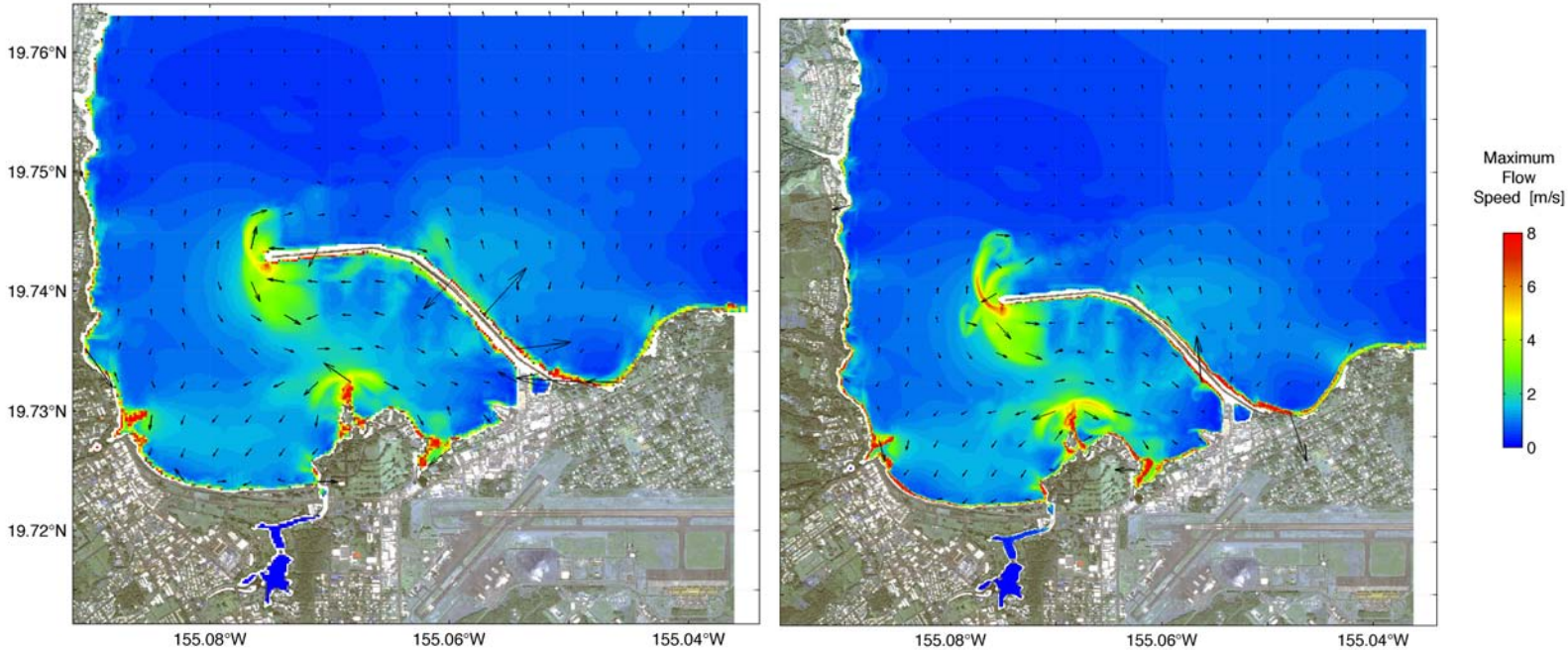
$\Delta x = 20\ m$ $\Delta x = 10\ m$ $\Delta x = 5\ m$

HA1126 - velocity



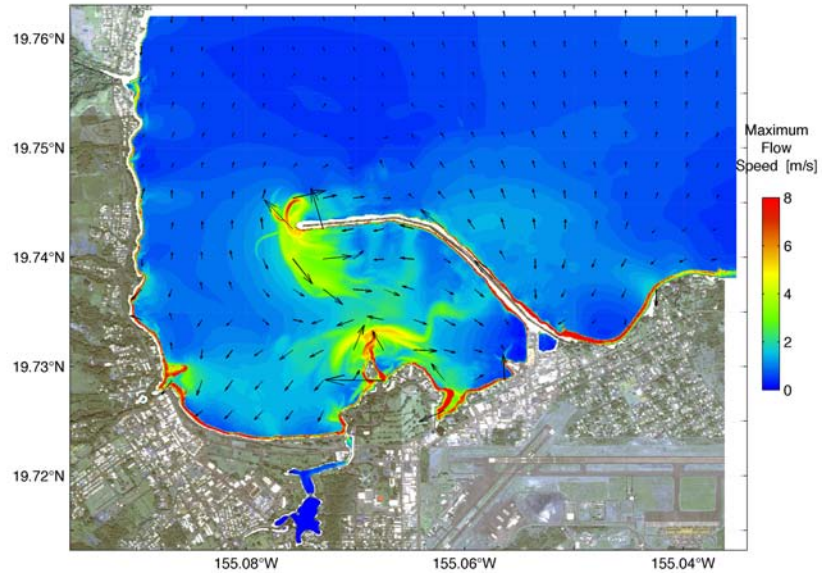
$\Delta x = 20\ m$ $\Delta x = 10\ m$ $\Delta x = 5\ m$

Maximum Velocities



dx = 20 m

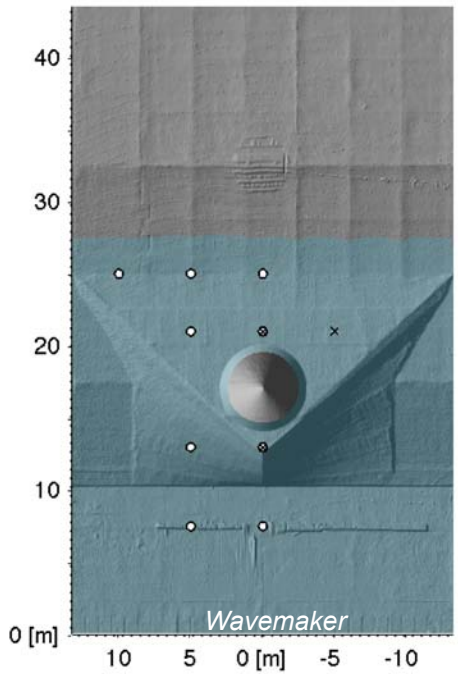
dx = 10 m

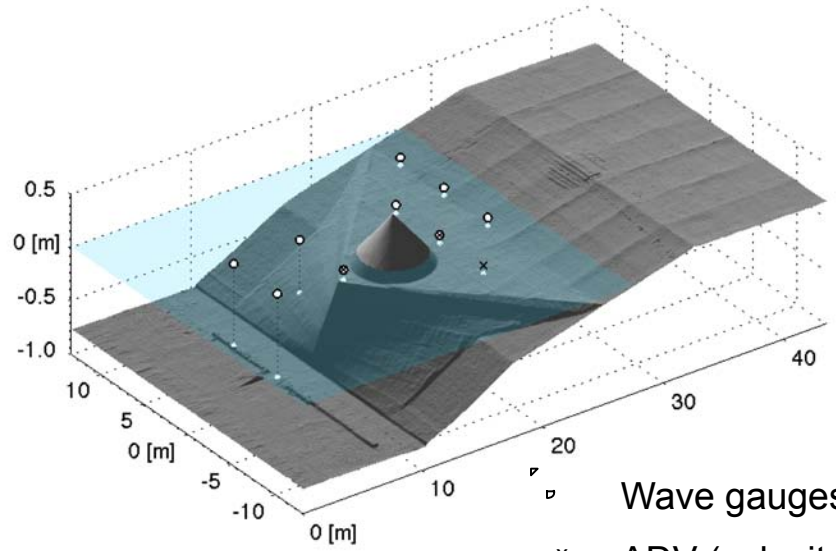


dx = 5 m

BM5, *reef with cone*

Top view





Wave gauges

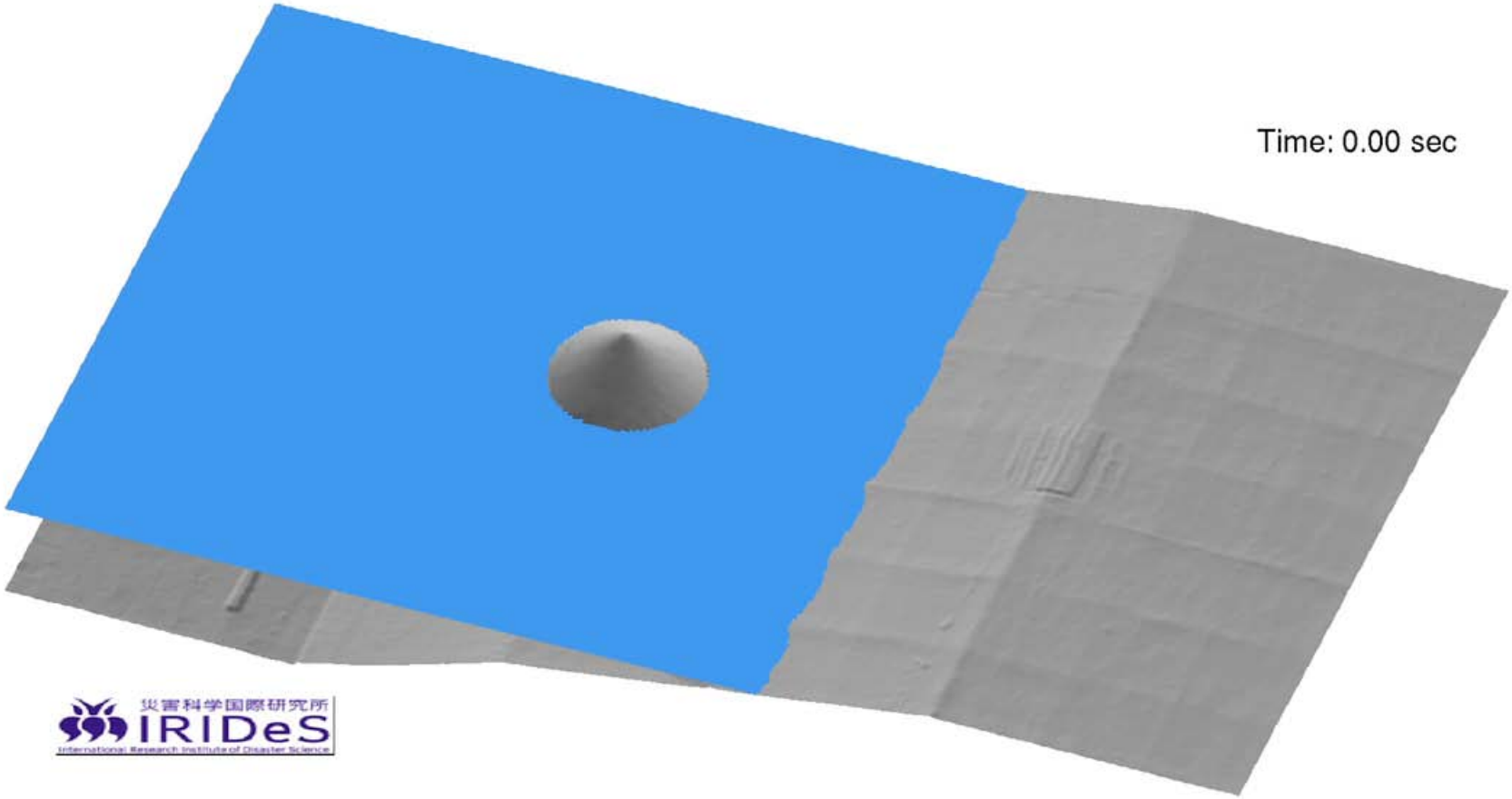
x ADV (velocity)

Laboratory Facility

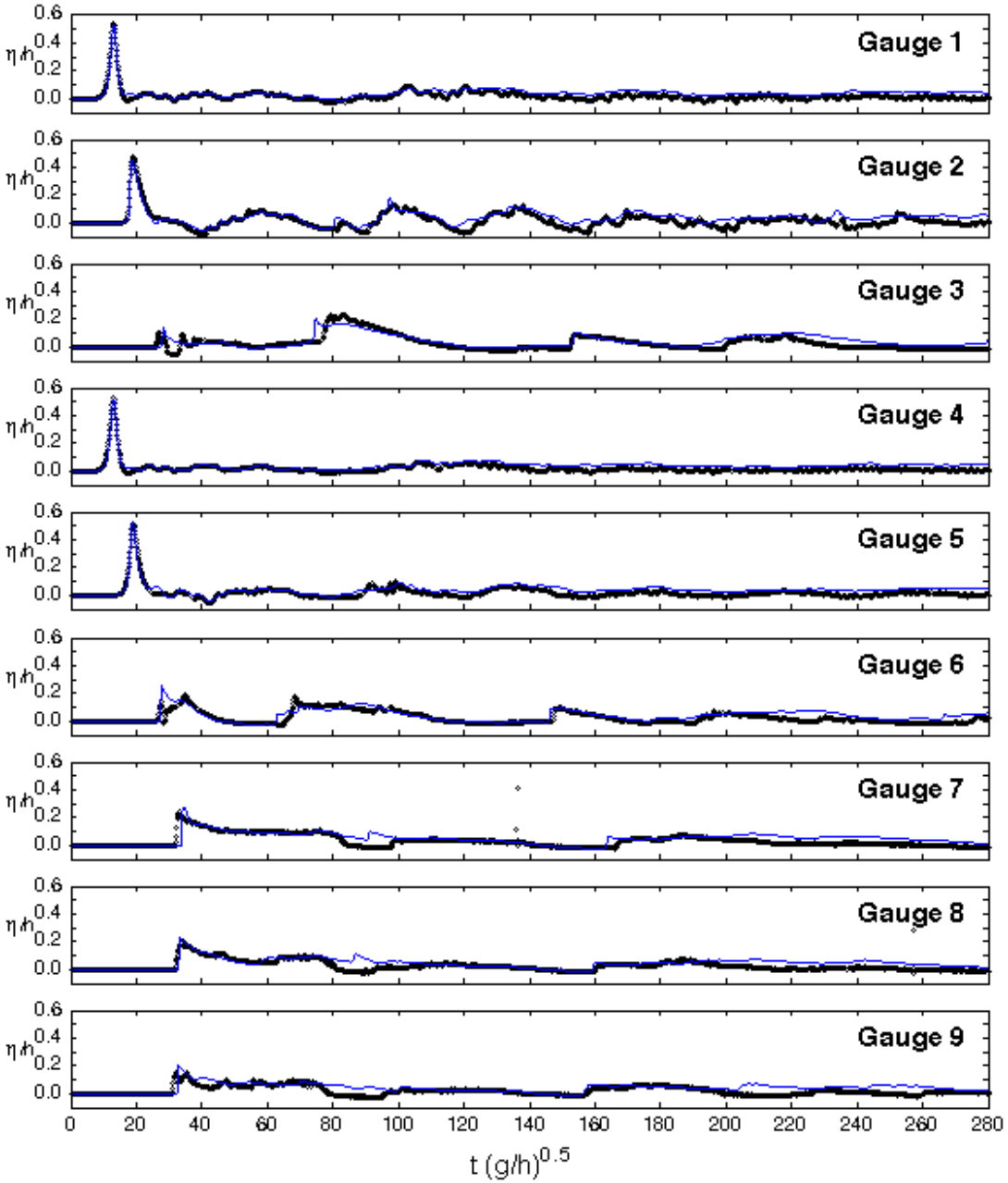
Tsunami Wave Basin,
Oregon State University

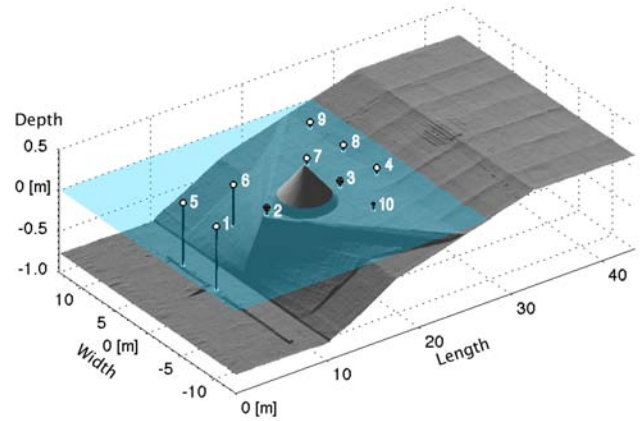
Tsunami Wave Basin
Wave height: **0.39** m
Water depth: **0.78** m
A/h: **0.5**
3D reef with apex

BOSZ - BM5



BM5 – free surface

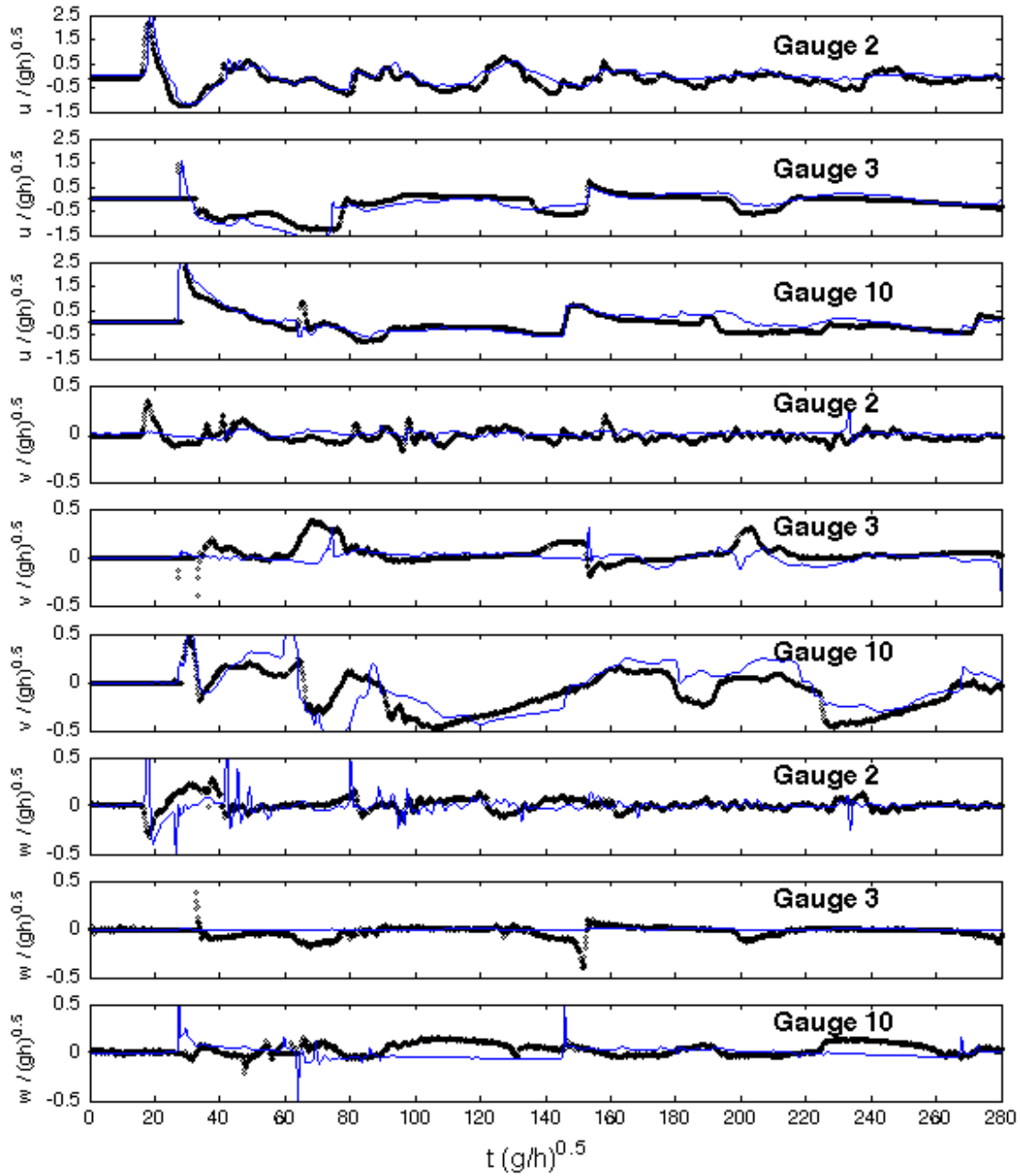


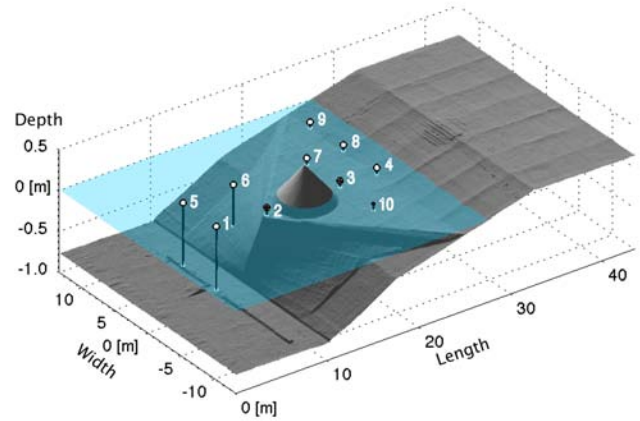


BOSZ

OSU data

BM5 – velocity





BOSZ

OSU data

Conclusions for BM tests

- Results are not very grid dependent.
- BOSZ resolves current flow structures reasonably well.
- Friction is a critical factor in shallow flows.
- Even for fairly hydrostatic problems, use dispersive model.

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