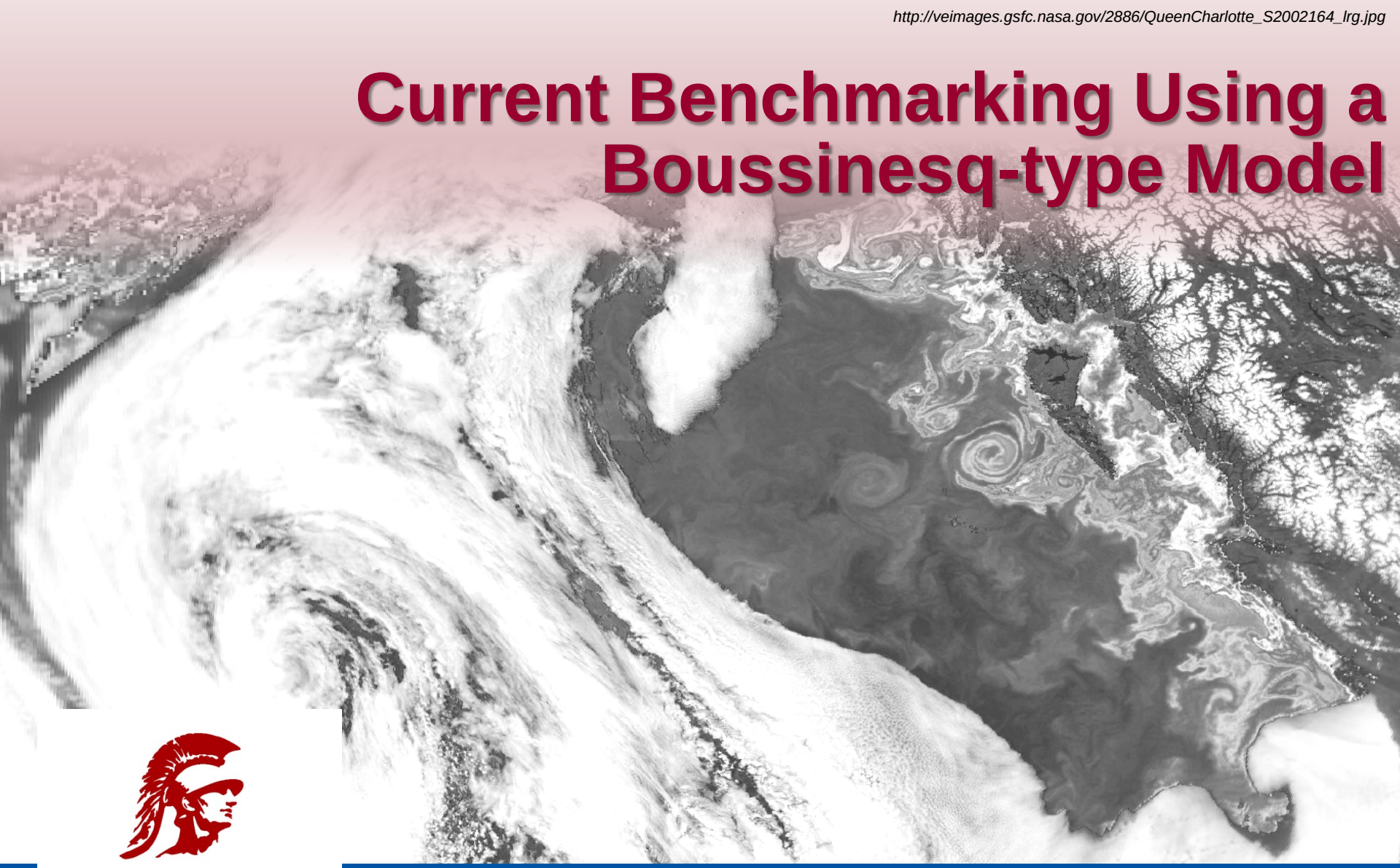
http://veimages.gsfc.nasa.gov/2886/QueenCharlotte_S2002164_lrg.jpg

Current Benchmarking Using a Boussinesq-type Model



Patrick Lynett

University of Southern California

Inclusion of Rotational & Turbulent Effects in Depth-Integrated Models

- *Theory: Kim et al. (2009, Ocean Modelling); Kim & Lynett (2011, Physics of Fluids)*

$O(\mu^2)$ Dispersive Corrections

$O(\beta\mu)$ Turbulent-Rotational Corrections

O(1) Shallow Water terms

$$\frac{\partial HU_i}{\partial t} + \frac{\partial HU_iU_j}{\partial x_j} + gH\frac{\partial \zeta}{\partial x_i} + H\left(D_i + \overline{\xi_i} + D_i^{\nu} + \overline{\xi_i^{\nu}}\right) + U_i\left(\mathcal{M} + \mathcal{M}^{\nu}\right)$$

$$- H\frac{\partial}{\partial x_j}\left(2\nu_t^h S_{ij}\right) + 2H\frac{\partial}{\partial x_i}\left(\nu_t^v \frac{\partial U_j}{\partial x_j}\right) + \frac{\tau_i^b}{\rho} - HF_i = 0$$

$O(\alpha\mu)$ Turbulent Mixing in Horizontal Plane. Eddy viscosity closed with Smagorinsky model

$O(\beta\mu)$ Turbulent Mixing in Vertical Plane. Eddy viscosity closed with Elder's model

$O(\beta\mu)$ Bottom Stress, closed with Mannings, Moody, etc.

$O(\gamma)$ Depth-averaging stress terms, closed with BSM

Horizontal vorticity effects

- $$\frac{\partial \boldsymbol{U}_\alpha}{\partial t} + \boldsymbol{U}_\alpha \cdot \nabla \boldsymbol{U}_\alpha + \nabla\zeta + \mu^2\left(\boldsymbol{D} + \overline{\boldsymbol{\xi}}\right) + \beta\mu\left(\boldsymbol{D}^\nu + \overline{\boldsymbol{\xi}^\nu}\right)$$
$$- \alpha\mu\nabla\cdot\left(\nu_t^h \nabla \boldsymbol{U}_\alpha\right) + \beta\mu\nu_t^v \nabla S + \beta\mu\frac{\boldsymbol{\tau}_b}{\zeta + h}$$
$$= O\left(\mu^4, \alpha\mu^3, \beta\mu^3, \beta^2\mu^2\right)$$

- $$\frac{\partial \zeta}{\partial t} + \nabla\cdot\left\{(\zeta + h)\,\boldsymbol{U}_\alpha\right\} + \mu^2\mathcal{M} + \beta\mu\mathcal{M}^\nu = O\left(\mu^4, \beta^2\mu^2\right)$$

  - ν_t^h : ***Smagorinsky model*** (1963)

$$\nu_t^{h\prime} = C_s^2\Delta^2 h_o\sqrt{g h_o}\sqrt{\left(\frac{\partial u}{\partial z}\right)^2 + 2\mu^2\left(\frac{\partial u}{\partial x}\right)^2 + 2\mu^2\left(\frac{\partial w}{\partial z}\right)^2 + \cdots\cdots}$$

$$\nu_t^{h\prime} = \alpha h_o\sqrt{g h_o}\,\nu_t^h$$

$$\alpha = C_s^2\Delta^2 \qquad O(\mu^2) = O(\alpha\mu) \ll 1$$

Horizontal vorticity effects

- $$\frac{\partial \boldsymbol{U}_\alpha}{\partial t} + \boldsymbol{U}_\alpha \cdot \nabla \boldsymbol{U}_\alpha + \nabla\zeta + \mu^2\left(\boldsymbol{D} + \overline{\boldsymbol{\xi}}\right) + \beta\mu\left(\boldsymbol{D}^\nu + \overline{\boldsymbol{\xi}^\nu}\right)$$
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- $$\frac{\partial \zeta}{\partial t} + \nabla\cdot\left\{(\zeta + h)\,\boldsymbol{U}_\alpha\right\} + \mu^2\mathcal{M} + \beta\mu\mathcal{M}^\nu = O\left(\mu^4, \beta^2\mu^2\right)$$

  - $\nu_t^v = \frac{\kappa}{6} H u_\tau$: ***Elder*** (1959)

$$\nu_t^{v\prime} = C_h H' u_*' \qquad u_* = C_* u_b$$

$$\nu_t^{v\prime} = \beta h_o\sqrt{g h_o} H u_b = \beta h_o \sqrt{g h_o}\,\nu_t^v$$

$$\beta = C_h C_*$$

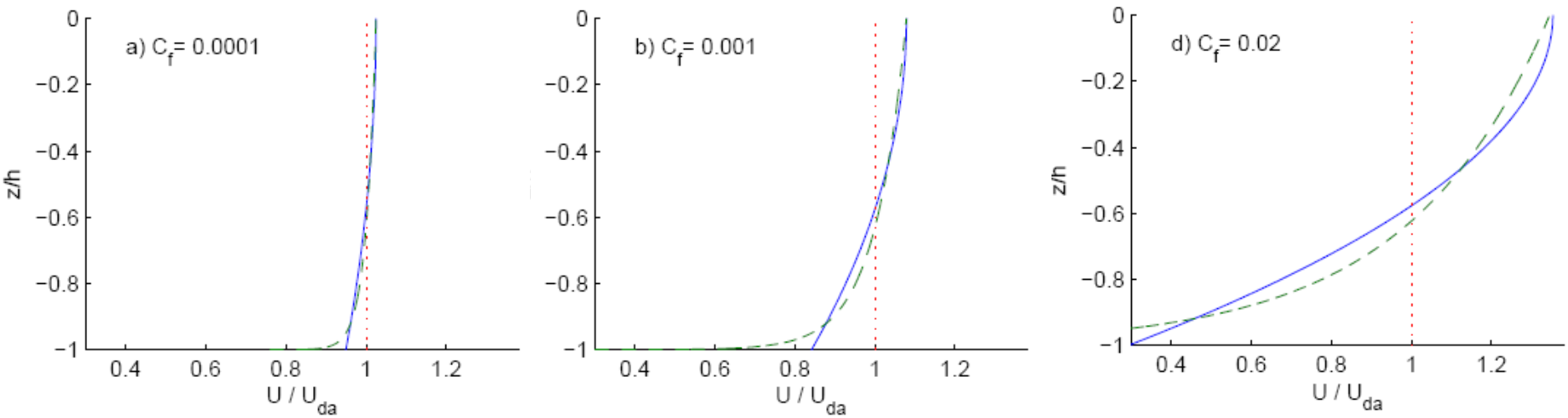
$$O(\mu^2) = O(\beta\mu) \ll 1$$

$$\boxed{\frac{\partial p}{\partial z} + 1 = O(\mu^2, \mu\beta)}$$

Horizontal vorticity effects

$$\boldsymbol{U} = \boldsymbol{U}_\alpha + \mu^2 \boldsymbol{U}_1^\phi + \beta\mu \boldsymbol{U}_1^r + O\left(\mu^4, \beta^2\mu^2\right)$$

$$\boldsymbol{U}_1^r = \int_{z_\alpha}^{z} \boldsymbol{\omega}_1 dz = \frac{\boldsymbol{\tau}_b}{\nu_t^v\left(\zeta + h\right)}\left\{\frac{1}{2}\left(z_\alpha^2 - z^2\right) + \zeta\left(z - z_\alpha\right)\right\}$$



$$U(z) = U_\alpha + \frac{\tau_b}{2\nu_t^v h}\left(z_\alpha^2 - z^2\right) \quad \text{———}$$

$$\tau_b = \rho C_f U_{DA}^2 \qquad \nu_t^v = \frac{\kappa}{6} h u_*$$

$$U_{log}(z) = U_{max} + \frac{u_*}{\kappa} ln\left(\frac{z+h}{h}\right) \quad \text{- - - - -}$$

$$u_* = \sqrt{\frac{\tau_b}{\rho}} = U_{DA}\sqrt{C_f}$$

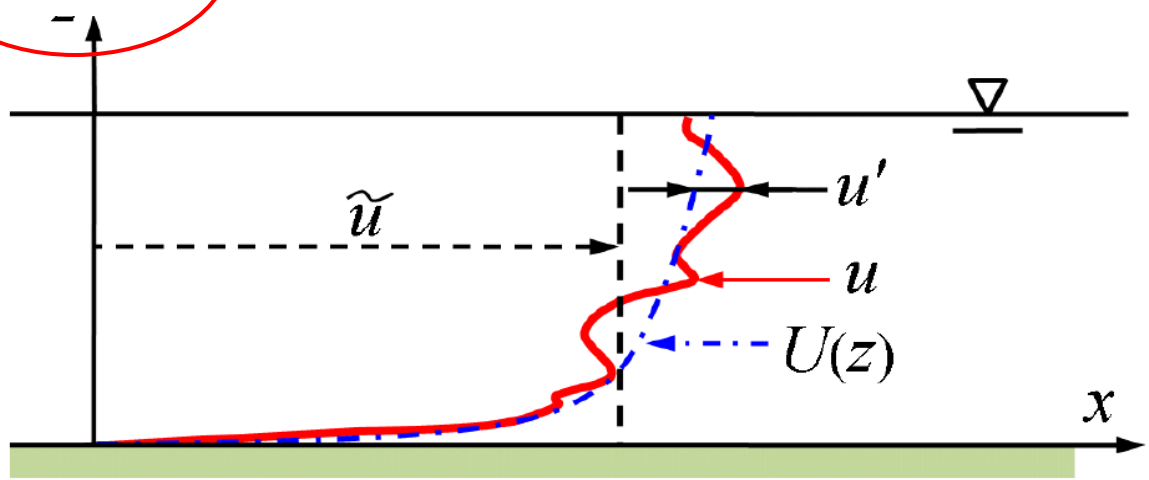
3D Turbulence Effects

- Go back to the beginning
  - Spatially filtered N-S equations
  - Depth-average

$$\frac{\partial H\widetilde{\overline{u}}_i}{\partial t} + \frac{\partial H\widetilde{\overline{u}}_i\widetilde{\overline{u}}_j}{\partial x_j} + H\widetilde{\frac{\partial \overline{p}}{\partial x_i}}$$
$$= \alpha\mu\frac{\partial}{\partial x_j}\left(2H\nu_t^h\widetilde{\overline{S_{ij}}}\right) + \beta\mu^2 2\nu_t^v\frac{\partial}{\partial x_i}\left(\frac{\partial u_j}{\partial x_j}\right) + \beta\mu^2\tau_i^b$$
$$+ \beta\mu^2\frac{\partial}{\partial x_j}\left(H\widetilde{\overline{u_i'u_j'}}\right)$$

$$F_i = C_B\frac{\sqrt{\widetilde{u}^2+\widetilde{v}^2}}{H}\sqrt{\frac{\nu\sqrt{c_f}}{\Delta t}}r_i$$

Stochastic BSM by ***Hinterberger, Frohlich, Rodi*** (2007)



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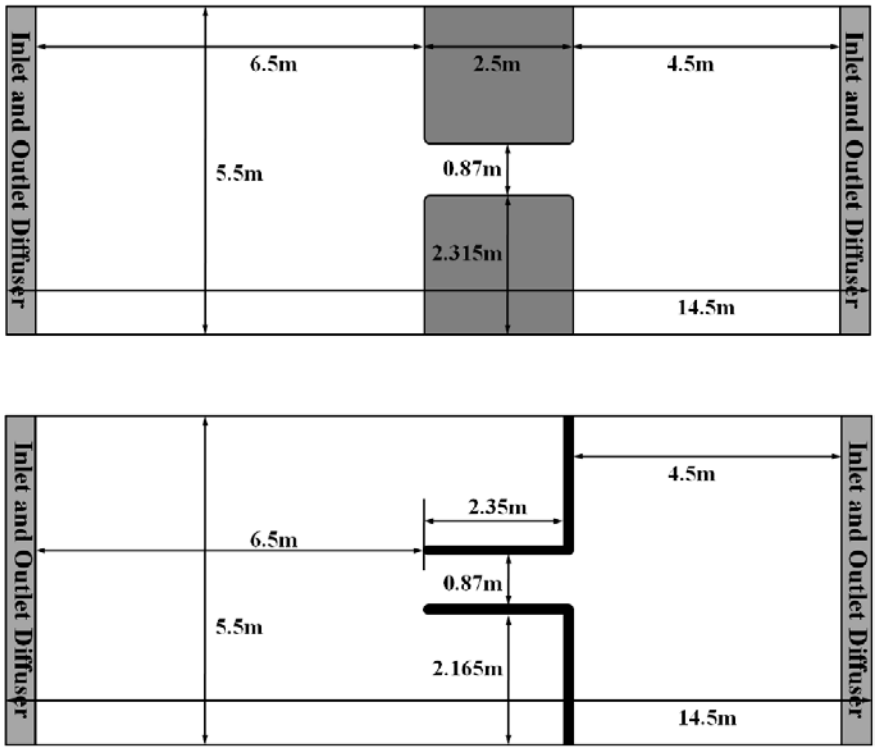
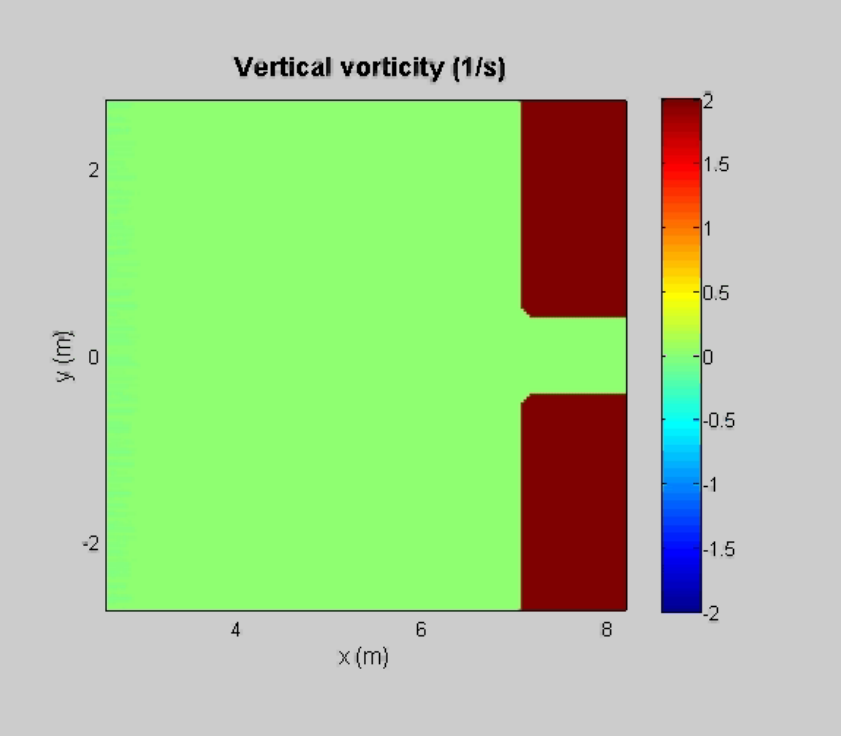
$O(\gamma)$ Depth-averaging stress terms, closed with BSM

Numerical Model

- Time integration :
  - 4th-order Predictor–Corrector scheme
- Leading-order term :
  - 4th-order MUSCL-TVD scheme, FVM
  - ***Yamamoto & Daiguji*** *(1993)*
- High-order term :
  - FVM discretization by ***Lacor et al***.(2004)
  - 4th-order or 2nd-order accuracy

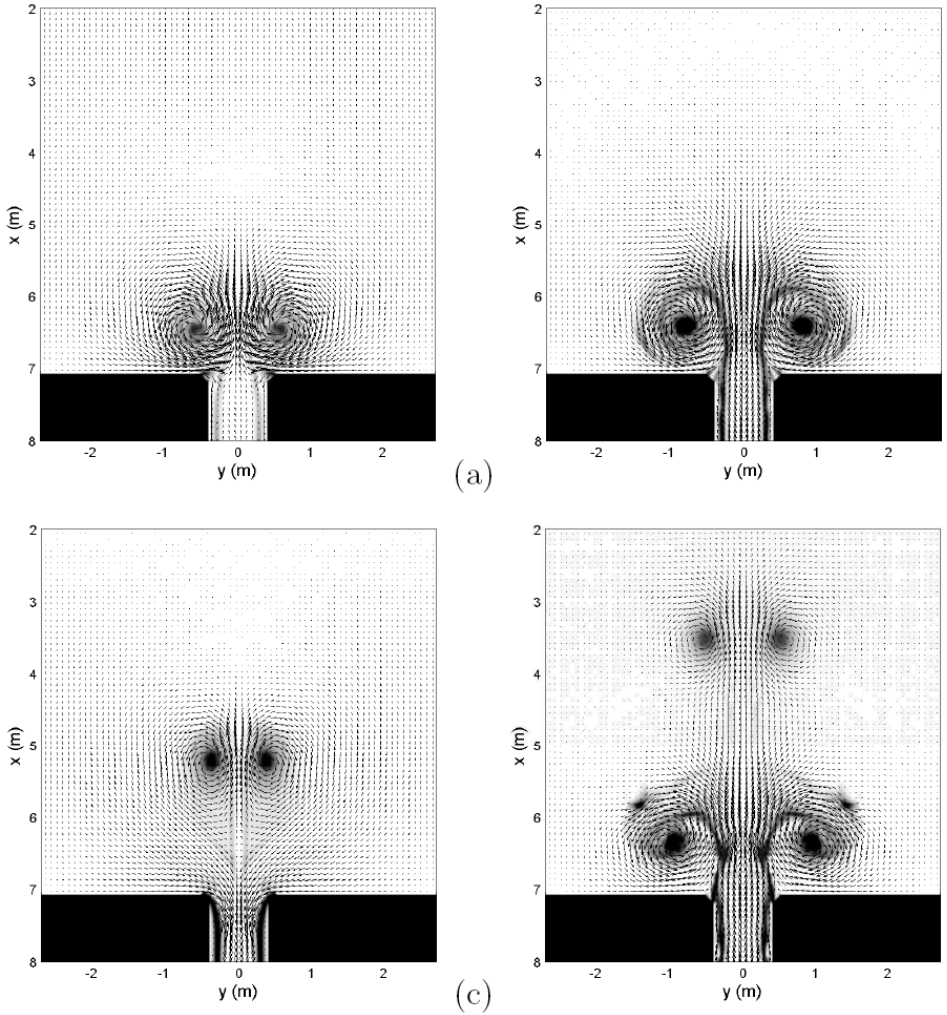
Coherent structures by tidal jet

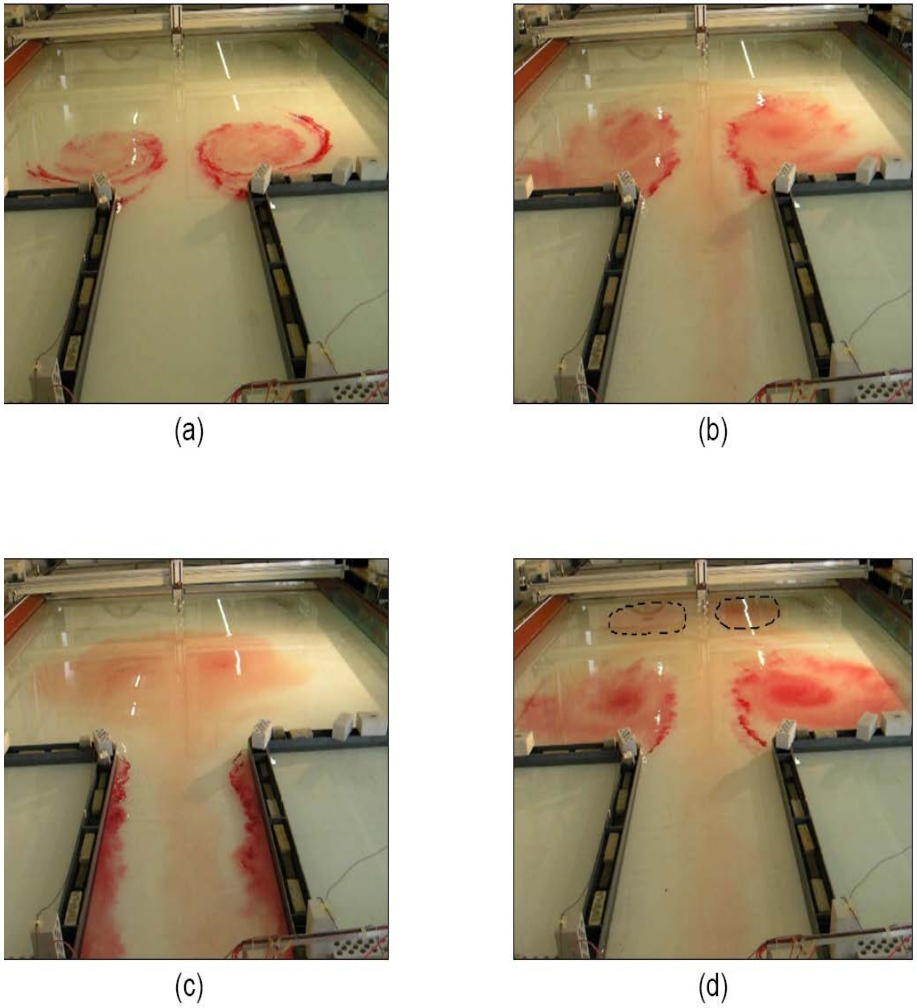




Experiment by ***Nicolau*** (2007)

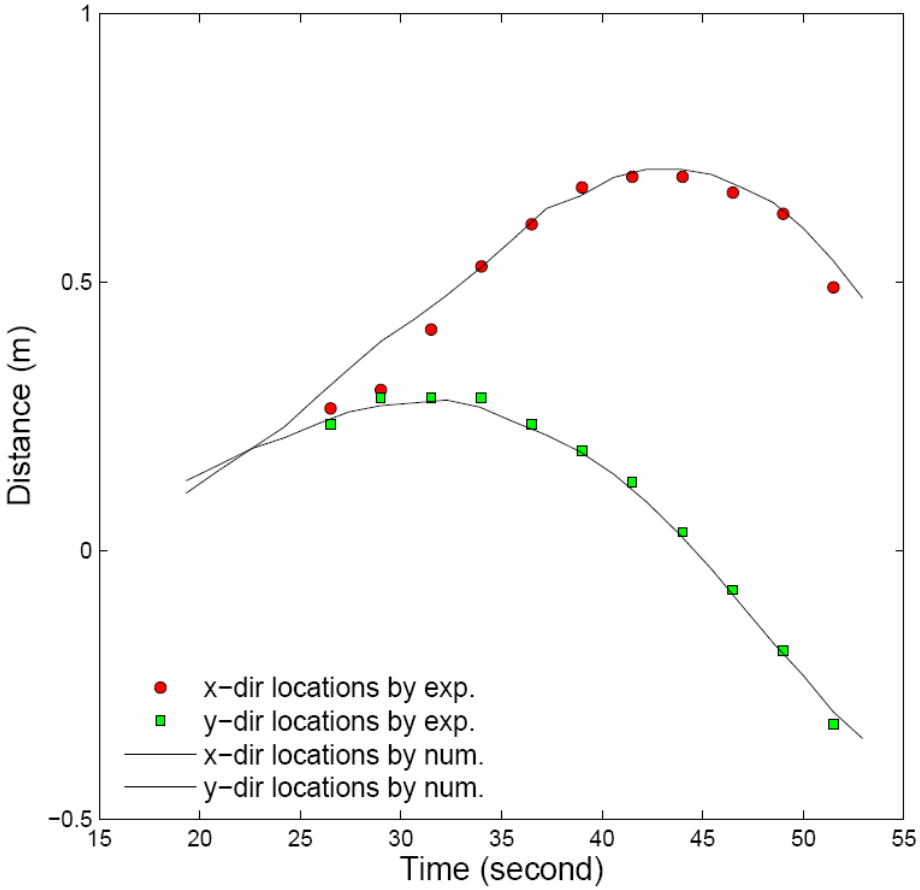
Coherent structure by tidal jet



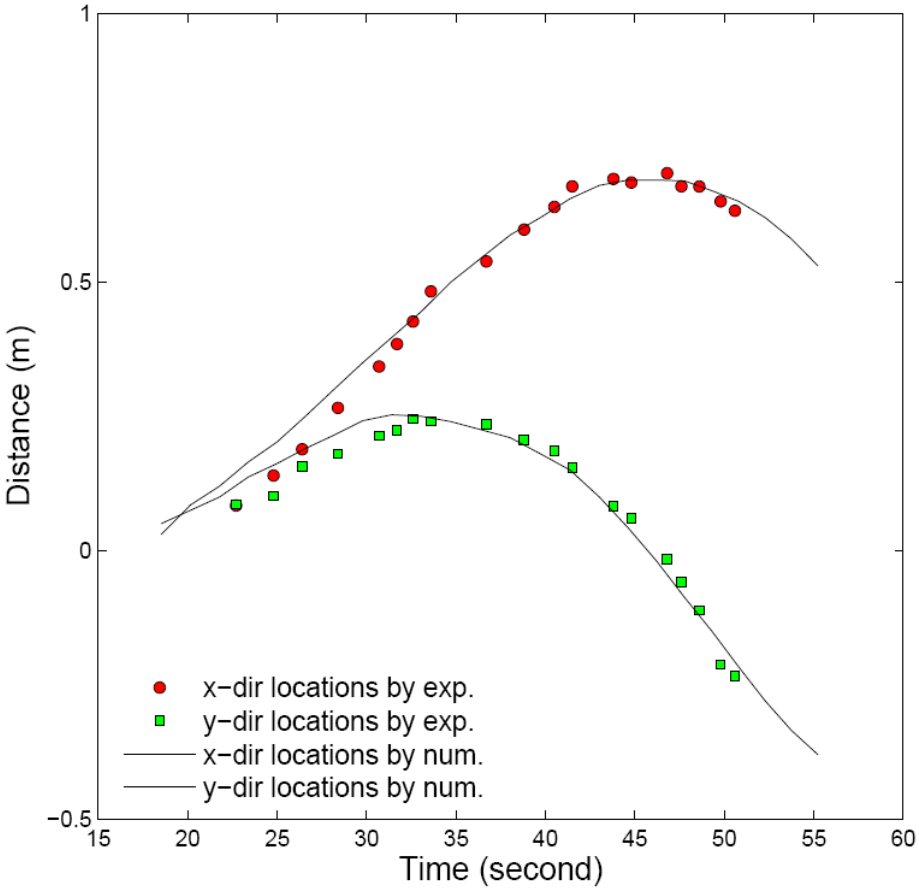


Experiment by ***Nicolau*** (2007)

Traces of vortex

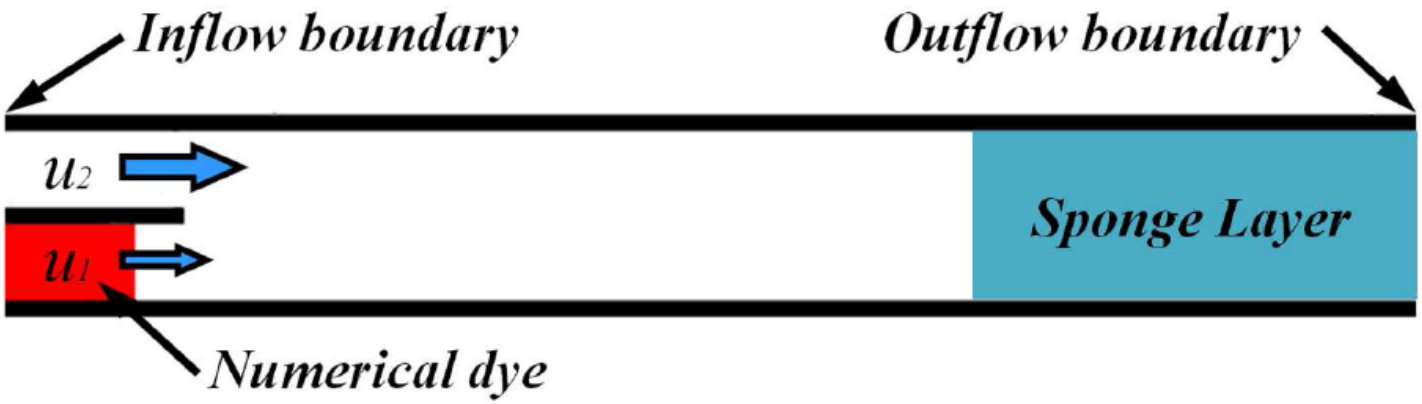


Layout D



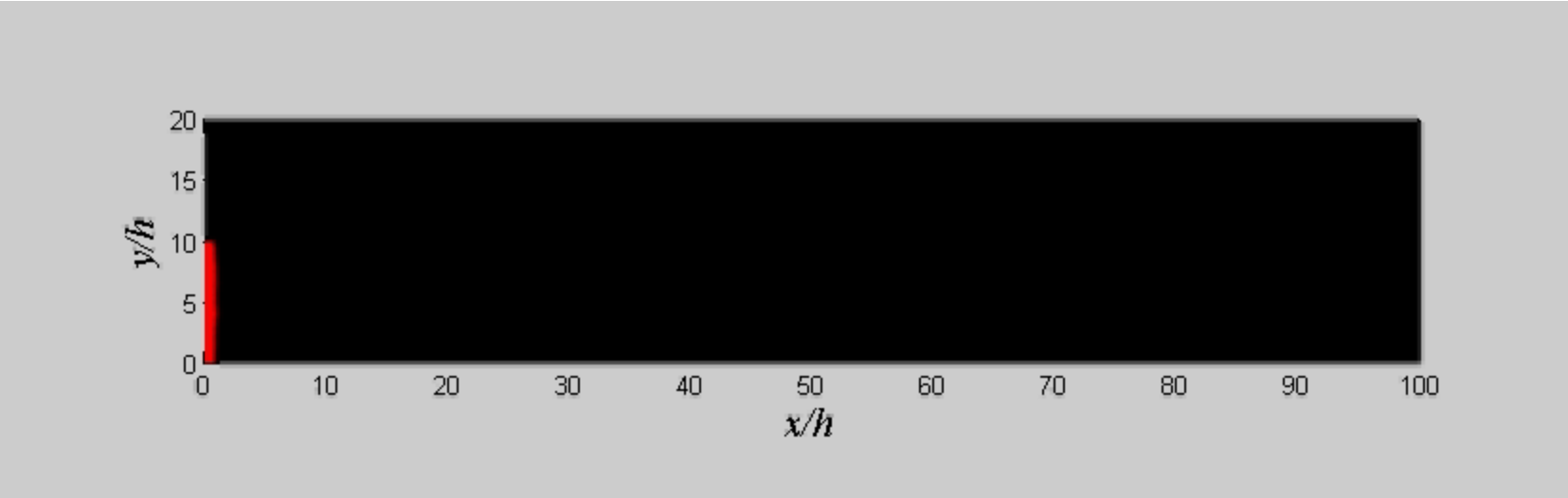
Layout C

Mixing by internal transverse shear instability

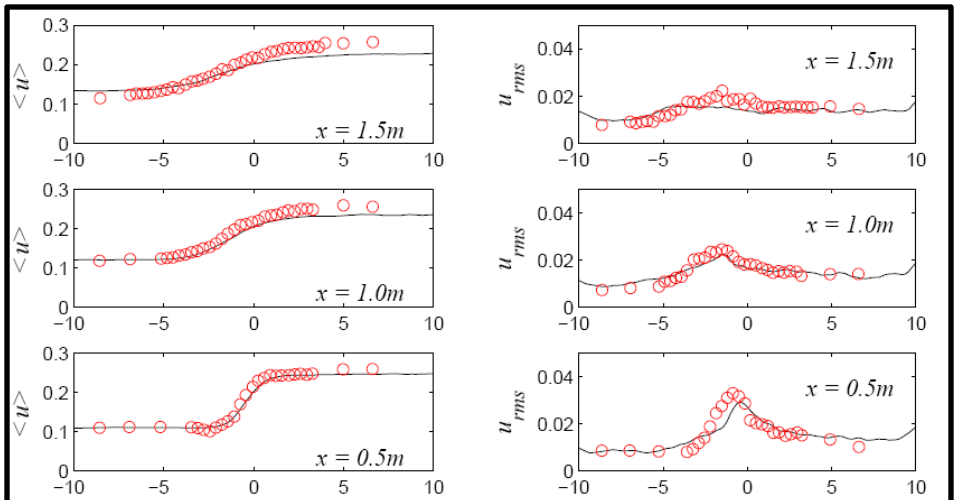


- u_1 = 0.111m/s, u_2 = 0.264m/s, Re = 5550

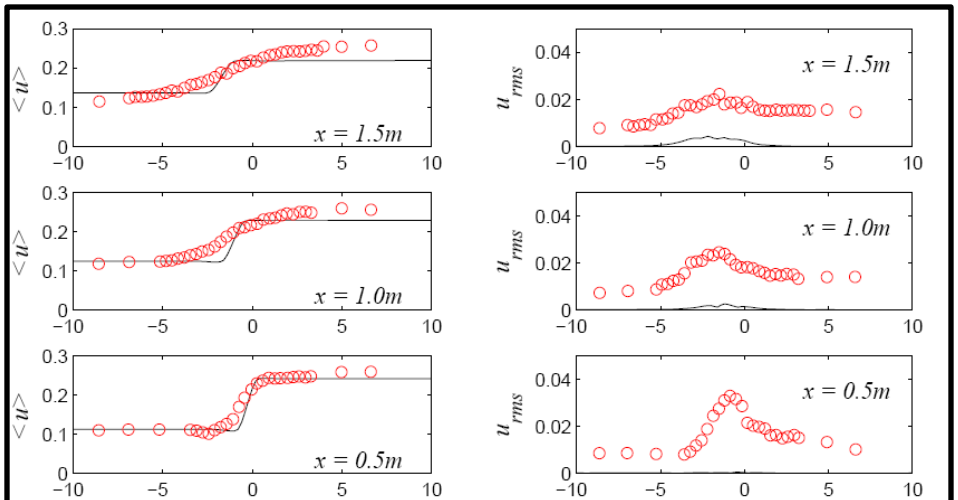
  Experiment by ***Babarutsi and Chu*** (1998)



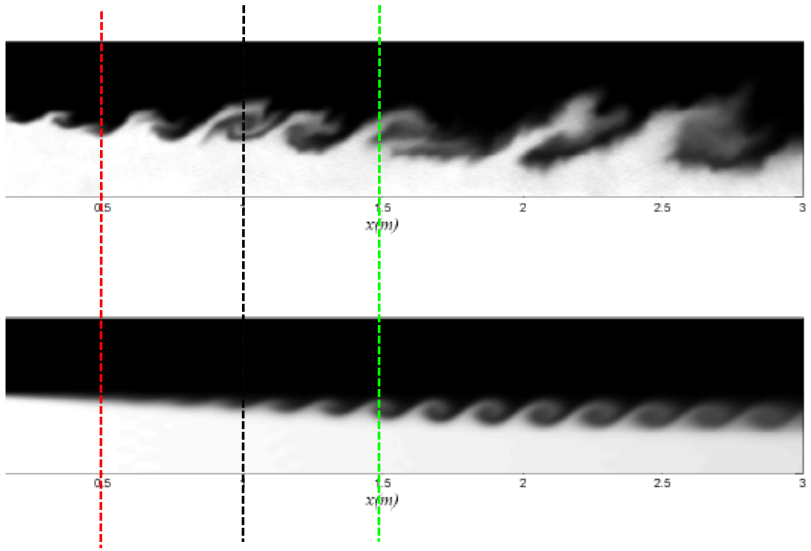
Energy transfer

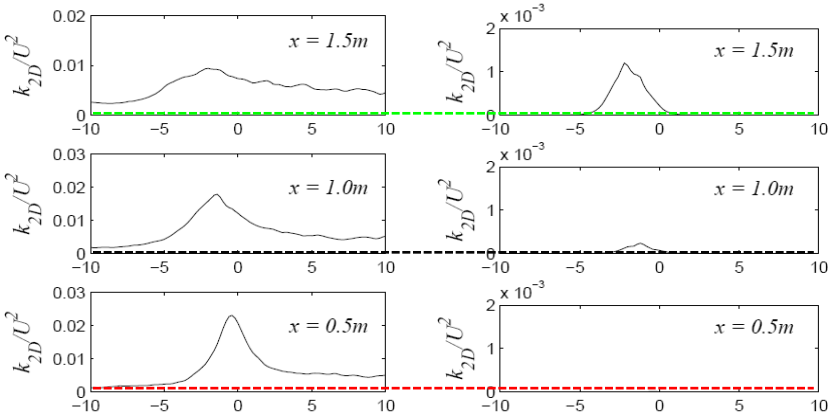


with BSM



without BSM





with BSM without BSM

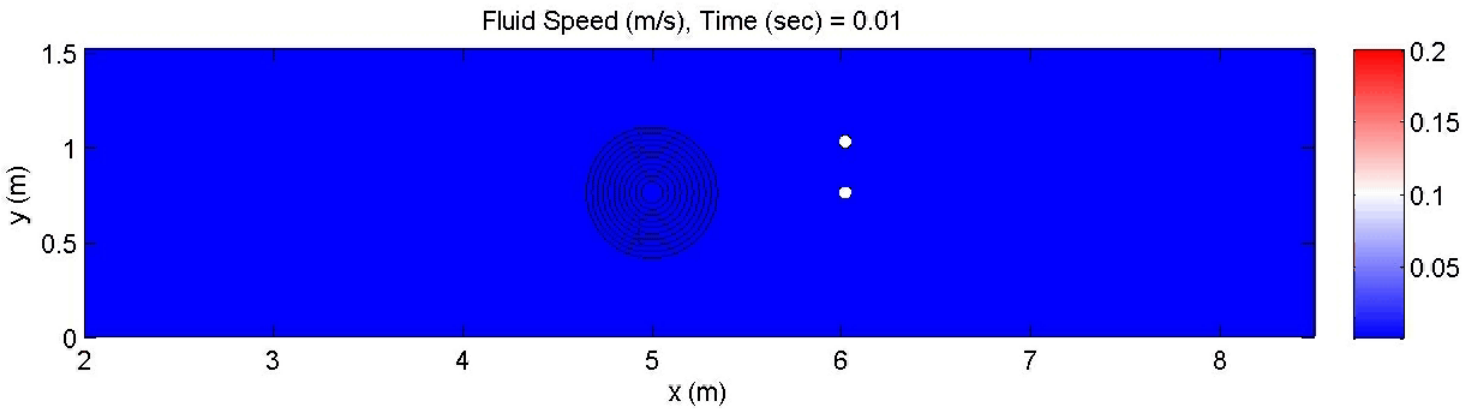
Benchmark #1

Run with different dissipation models

Resolution = 1 cm

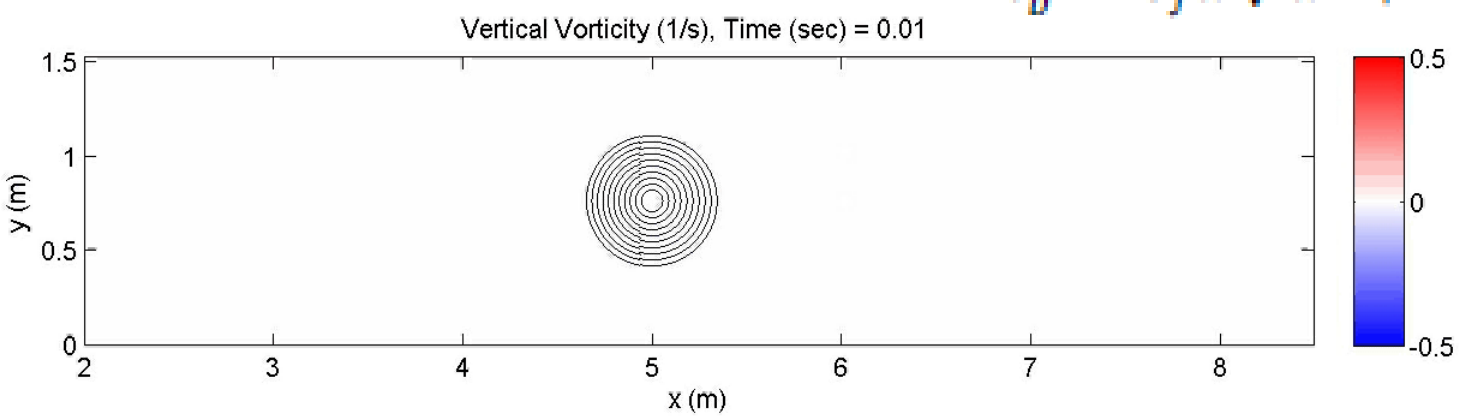
Roughness height = 0.015 mm (hydraulically smooth)

Smag coef = 0.2



The roughness coefficient Cf = f/4 (Chen and Jirka, 1995) and f is estimated using the Moody diagram, which here is calculated by the explicit formula given by Haaland (1983).

$$\tau_b^x = C_f u \sqrt{u^2 + v^2}$$



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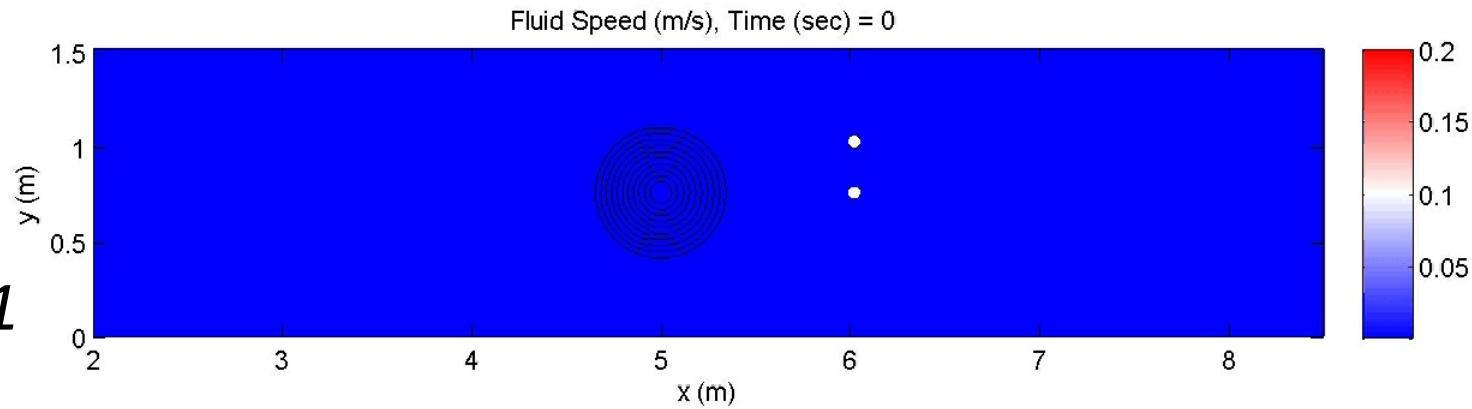

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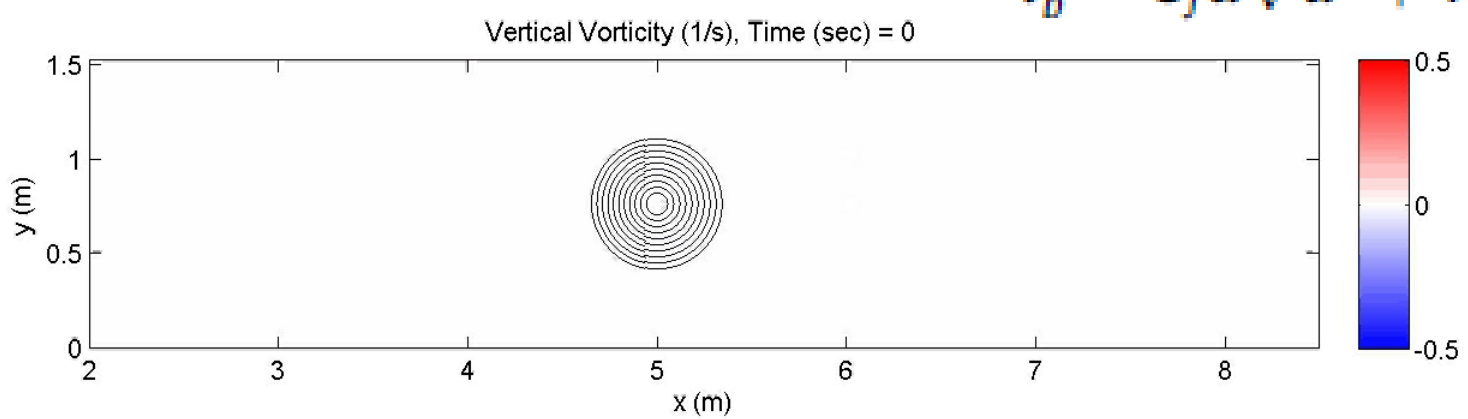
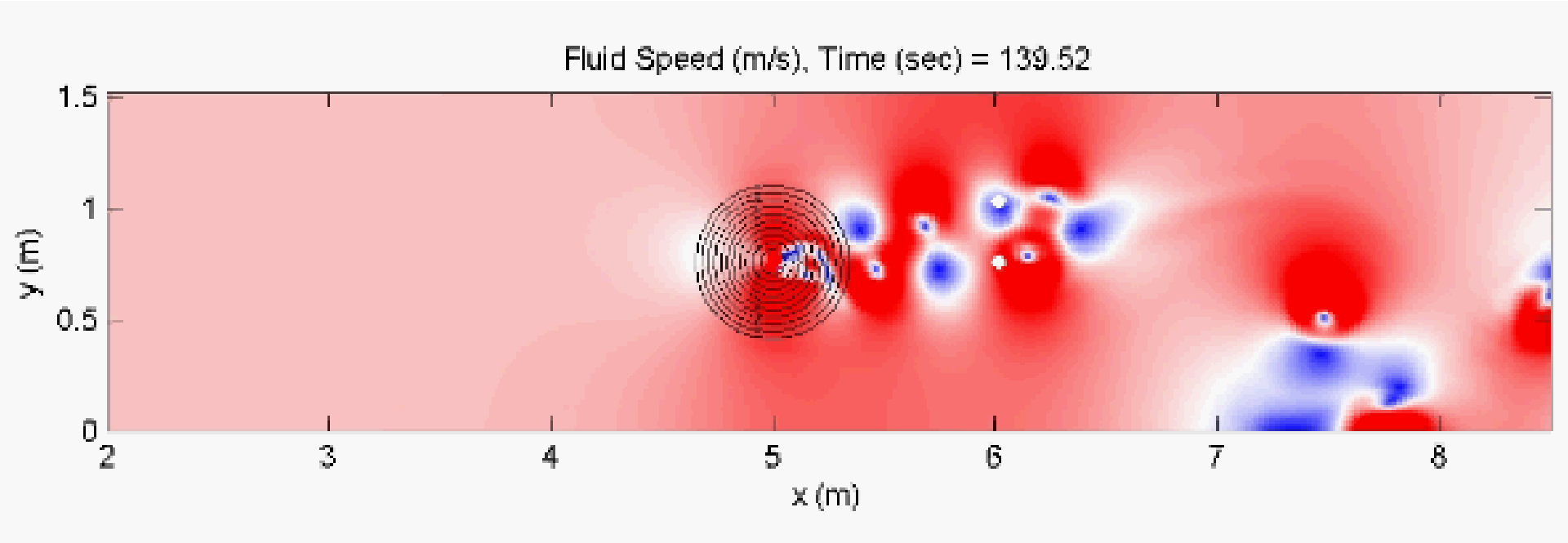


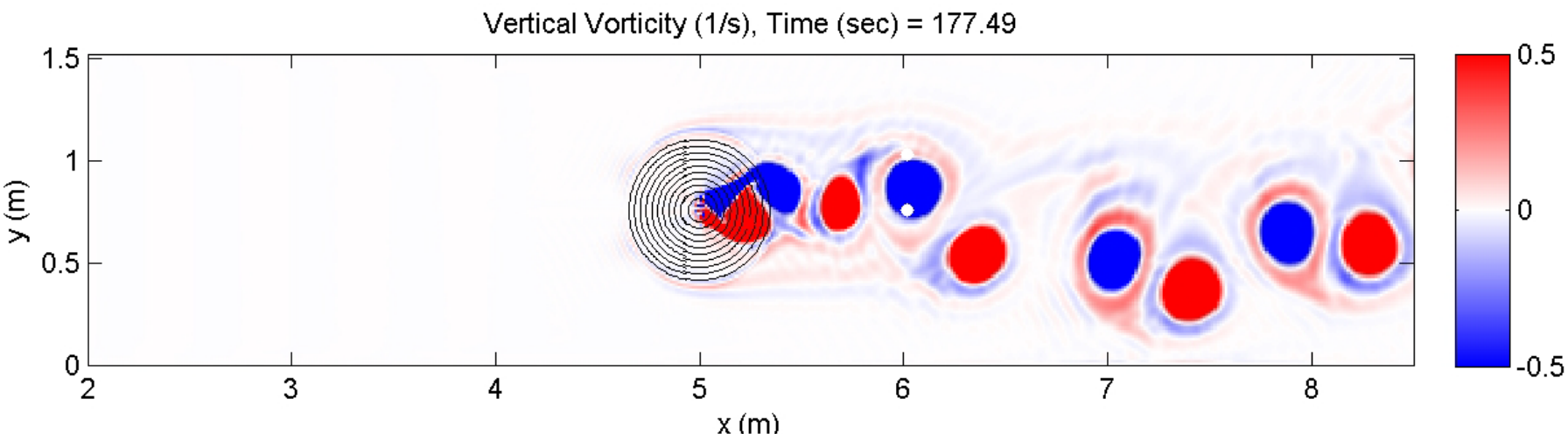
Benchmark #1

- *Simulations with all dissipation models off:*
  - With no limiters used, simulations crash due to instabilities at island apex, for resolutions smaller than 0.02 m
  - When using the minmod limiter (van Leer, 1979), stable results can be achieved to resolutions of 0.01 m, but no numerical convergence (in the deterministic sense) is found



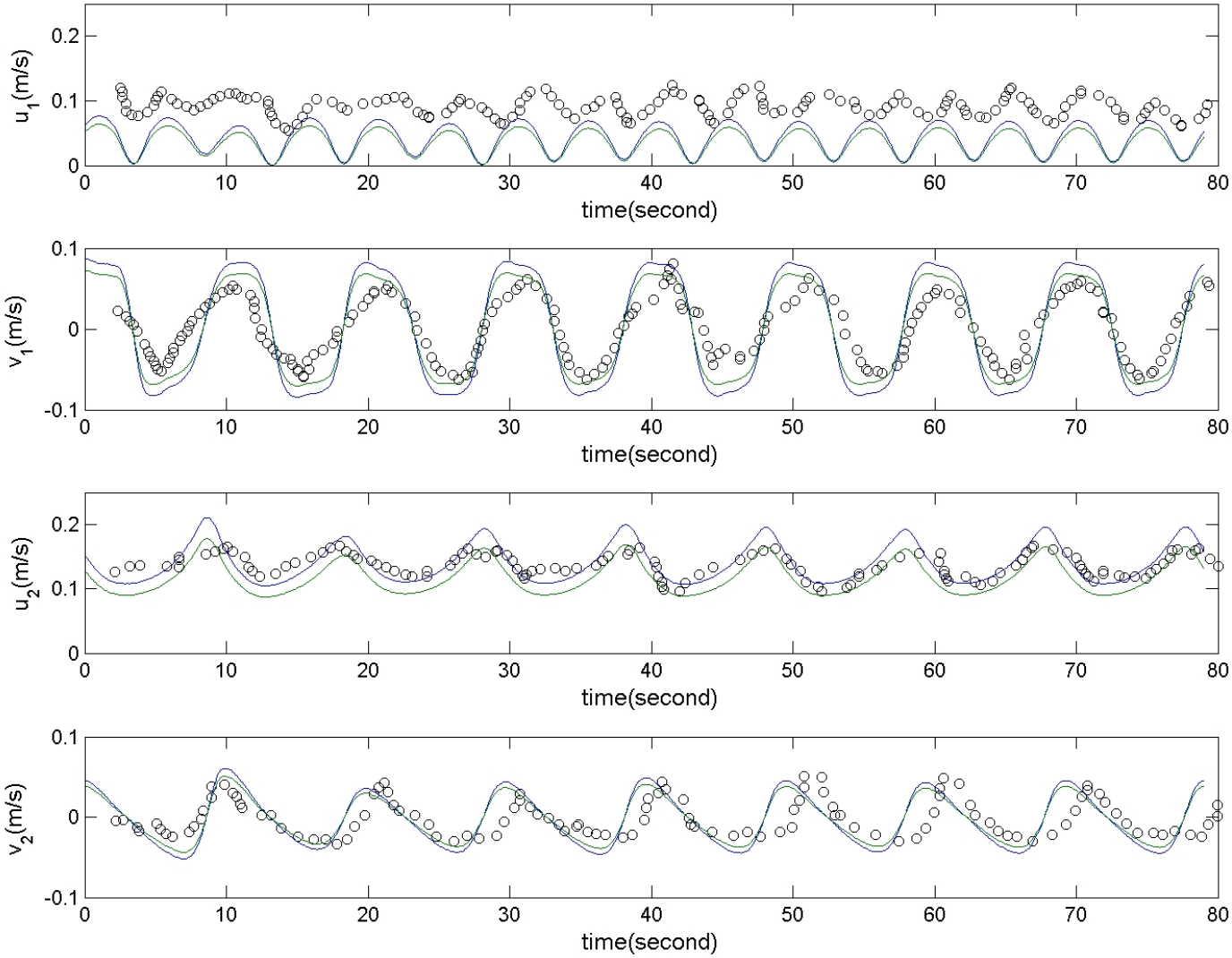
Benchmark #1

- *Simulations with prescribed bottom friction:*
  - Using the roughness height friction model, numerically convergent results (after spin-up) are found at a resolution of 0.015 m
    - Using the backscatter model, agreement with data is best
  - Numerical convergence (in the deterministic sense) was not found with prescribed Mannings or friction coefficient (chaotic wake) –



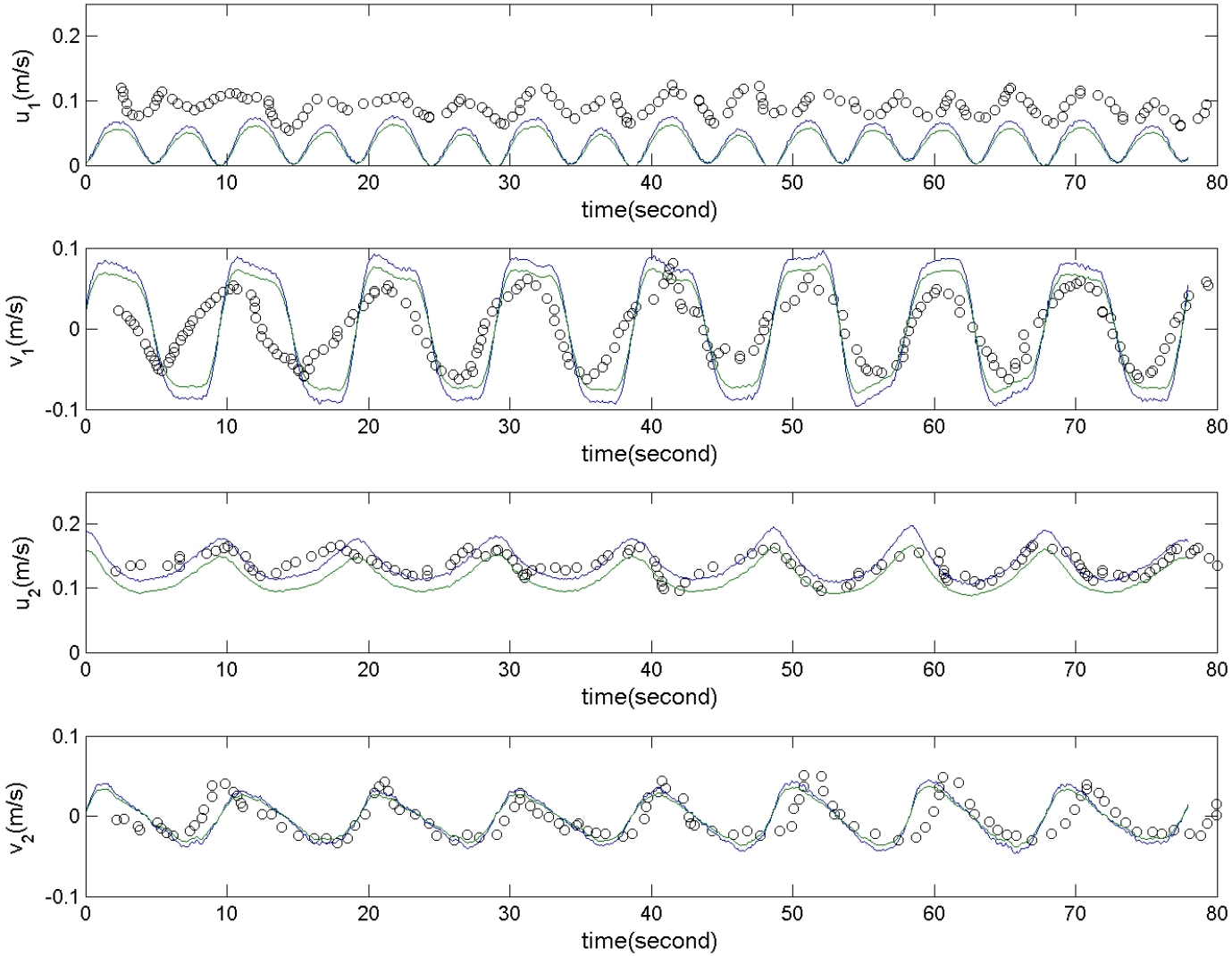
Benchmark #1

- *Roughness Height Model (No Backscatter)*



Benchmark #1

- *Roughness Height Model (WITH Backscatter)*

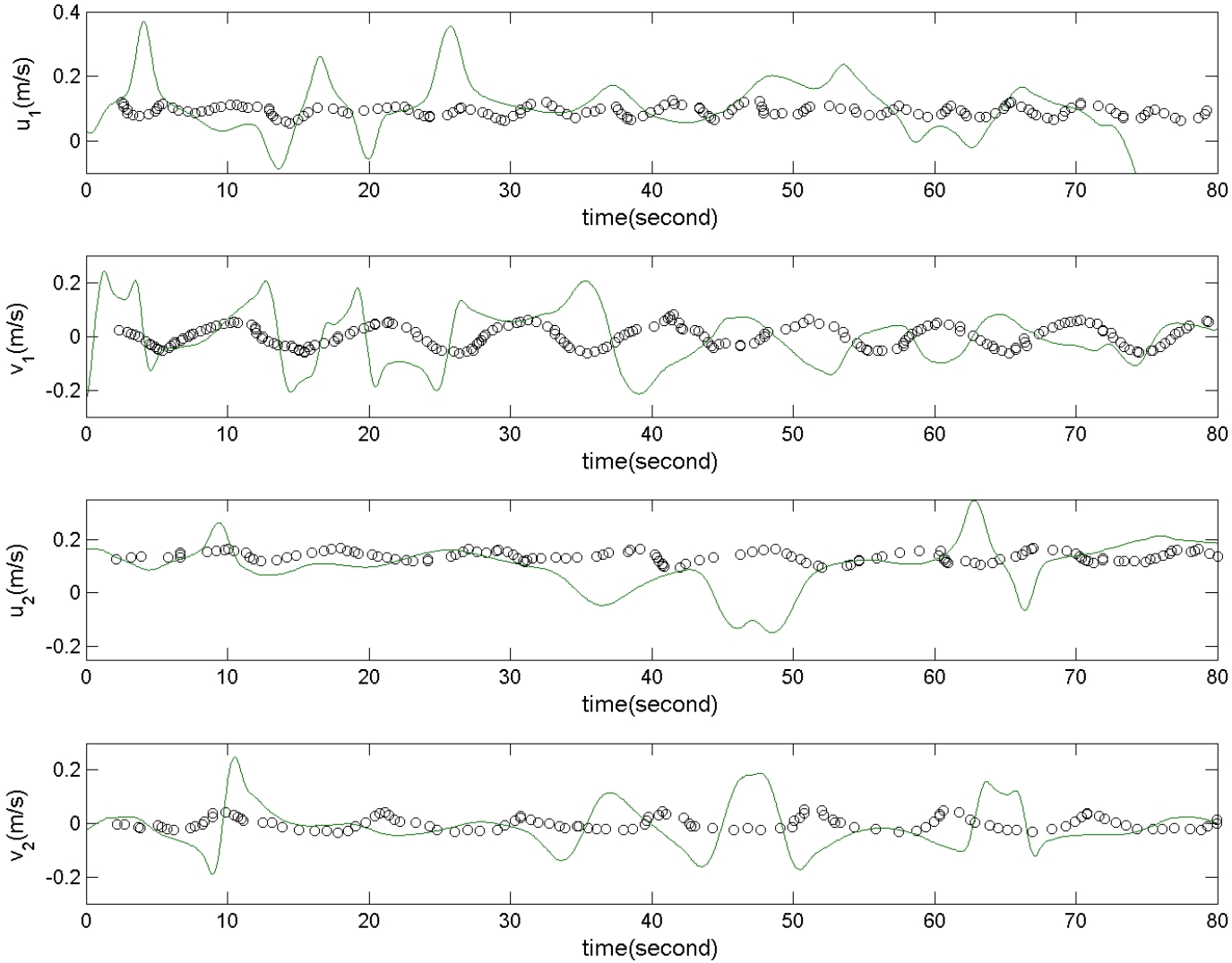


Benchmark #1

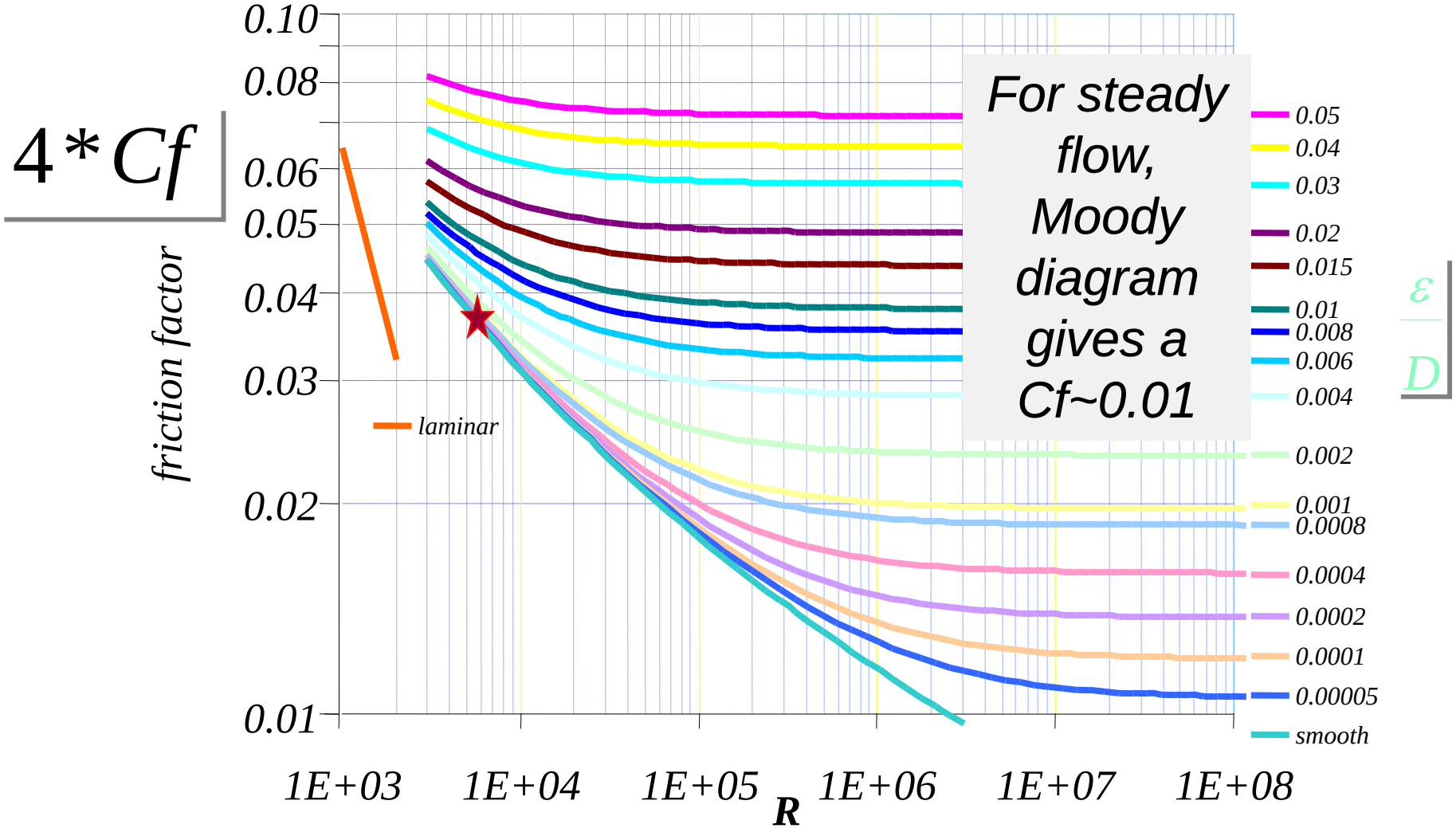
- *Constant Cf= 0.006 model*

Friction Factor formulation VERY Important

How do we know which model is appropriate on the geophysical scale?



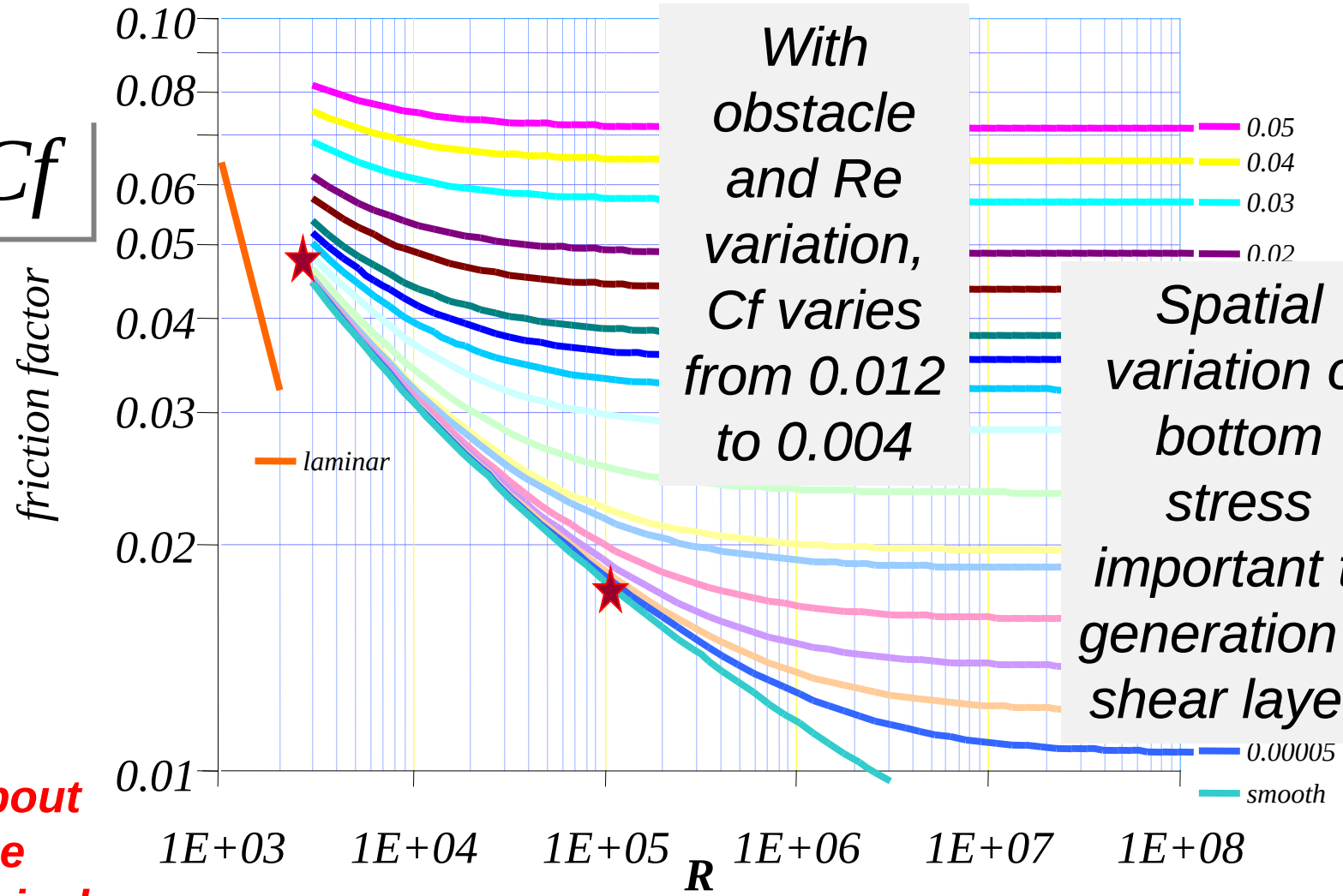
Moody Diagram



$$\tau_b^x = C_f u\sqrt{u^2+v^2}, \quad \tau_b^y = C_f v\sqrt{u^2,+v^2}$$

Moody Diagram

$4 * Cf$



With obstacle and Re variation, Cf varies from 0.012 to 0.004

Spatial variation of bottom stress important to generation of shear layers

What about on the geophysical scale?

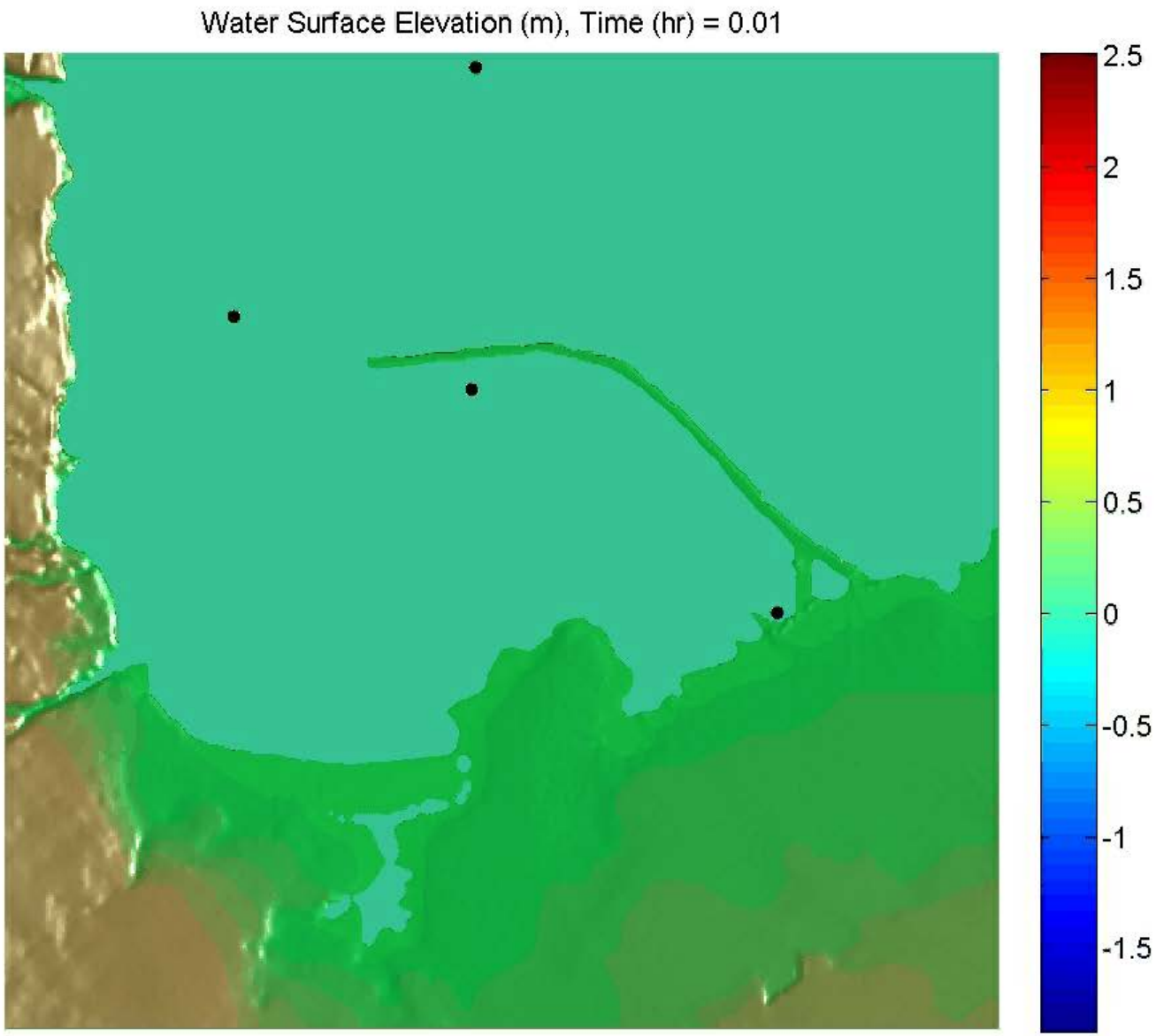
$$\tau_b^x = C_f u \sqrt{u^2 + v^2}, \quad \tau_b^y = C_f v \sqrt{u^2, +v^2}$$

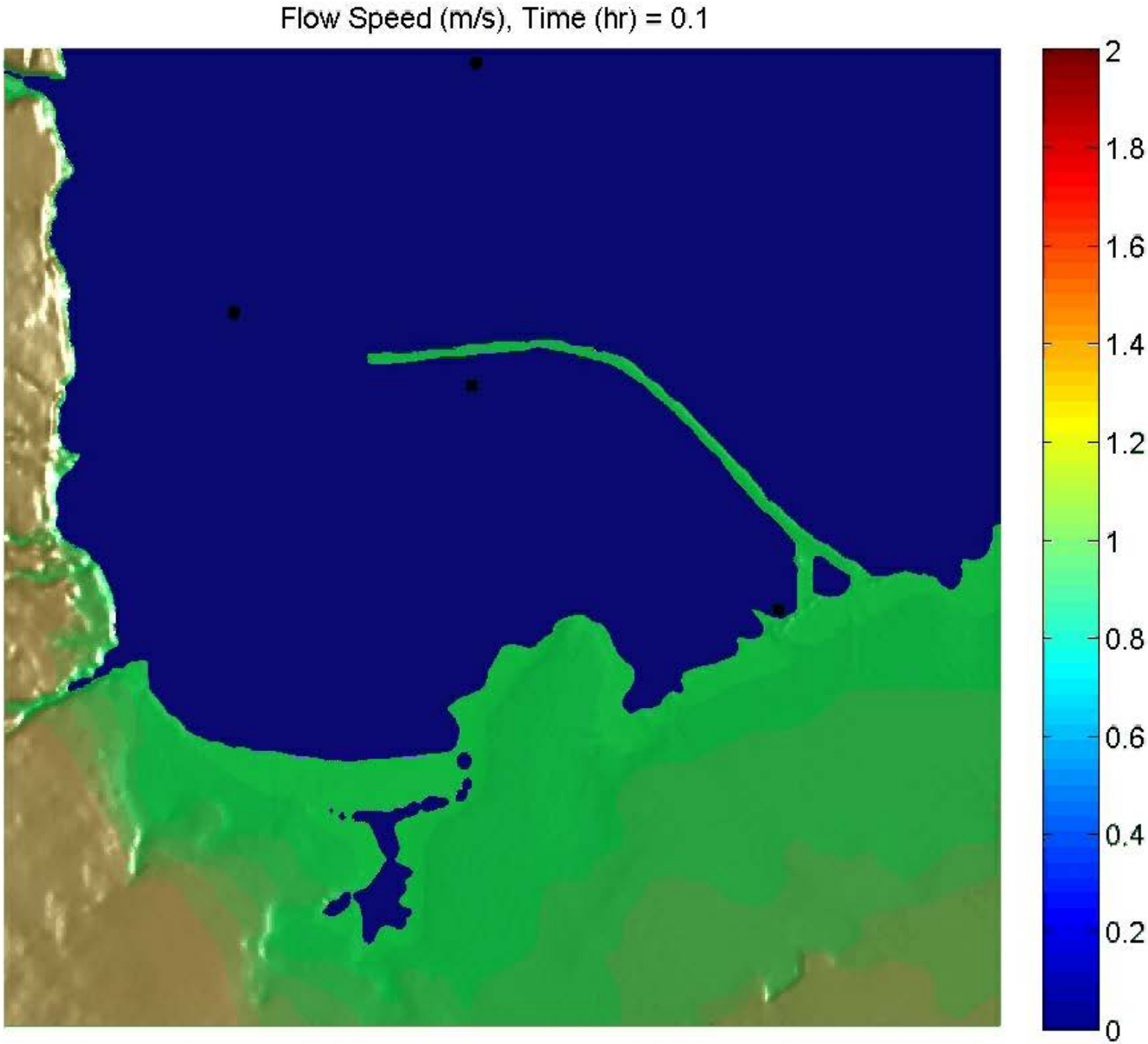
Benchmark #2

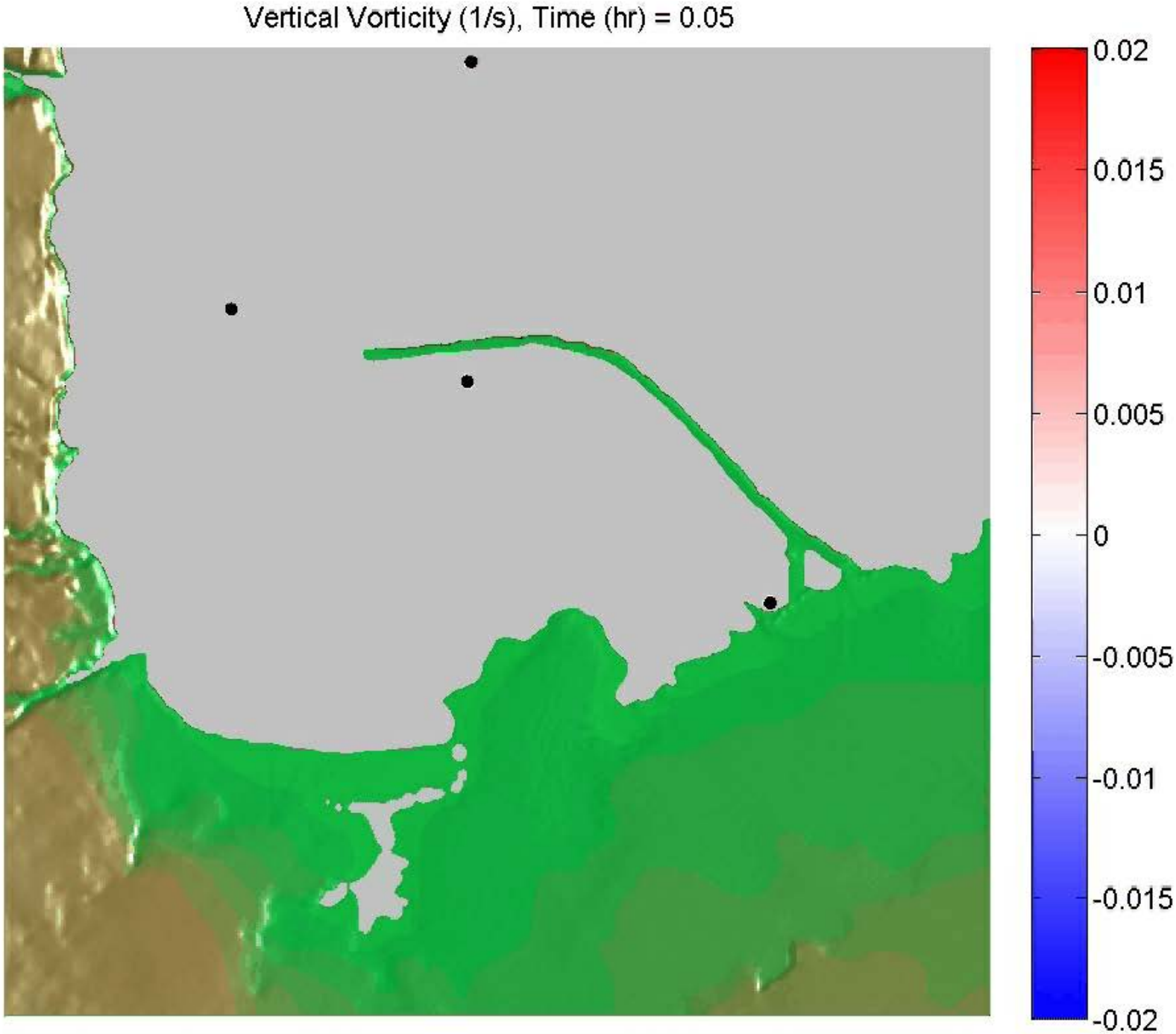
Run at three different resolutions (5m ,10m, 20m)

Mannings n constant at 0.025

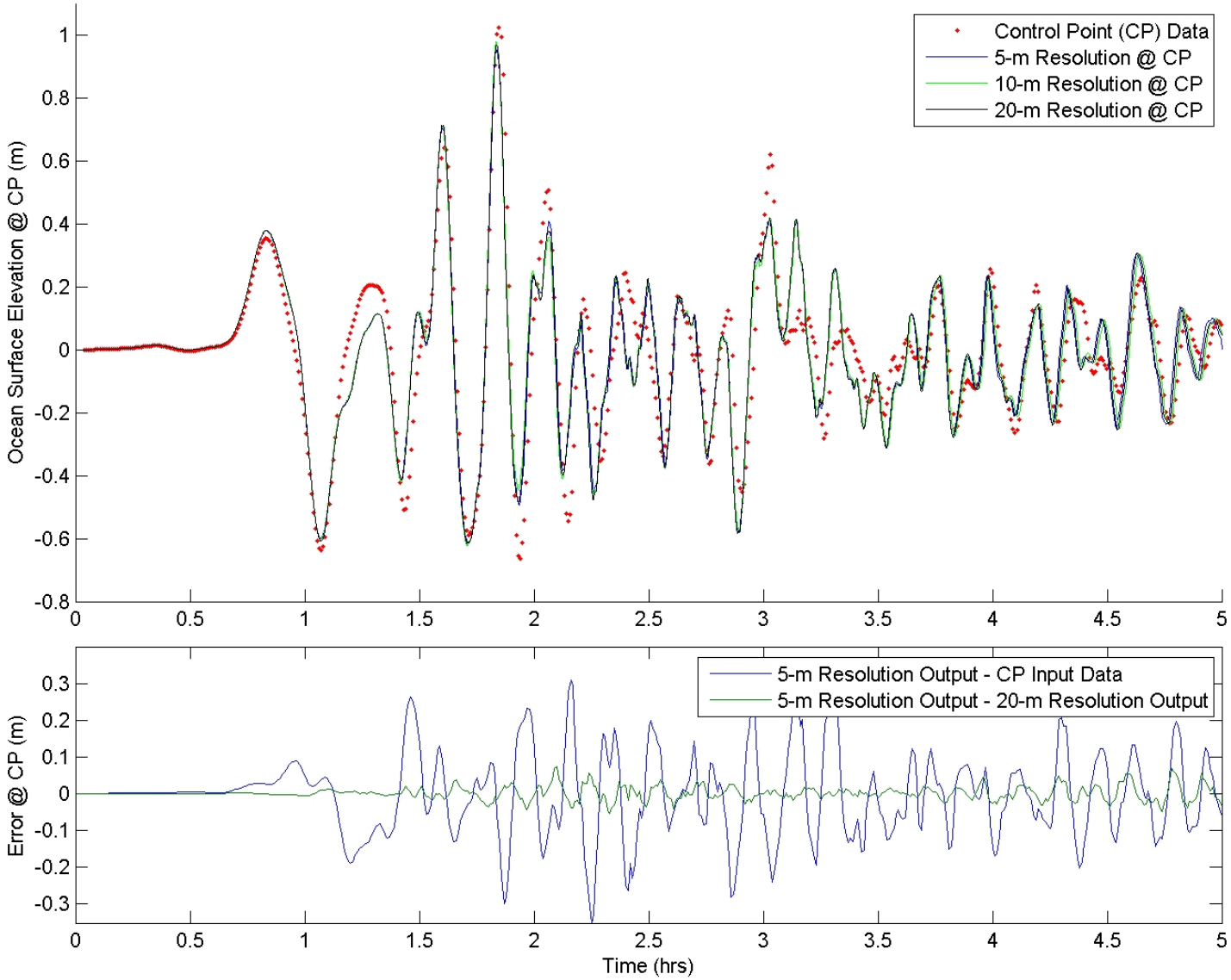
Smag coef = 0.2




Flow Speed (m/s), Time (hr) = 0.1
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0


Vertical Vorticity (1/s), Time (hr) = 0.05
0.02
0.015
0.01
0.005
0
-0.005
-0.01
-0.015
-0.02

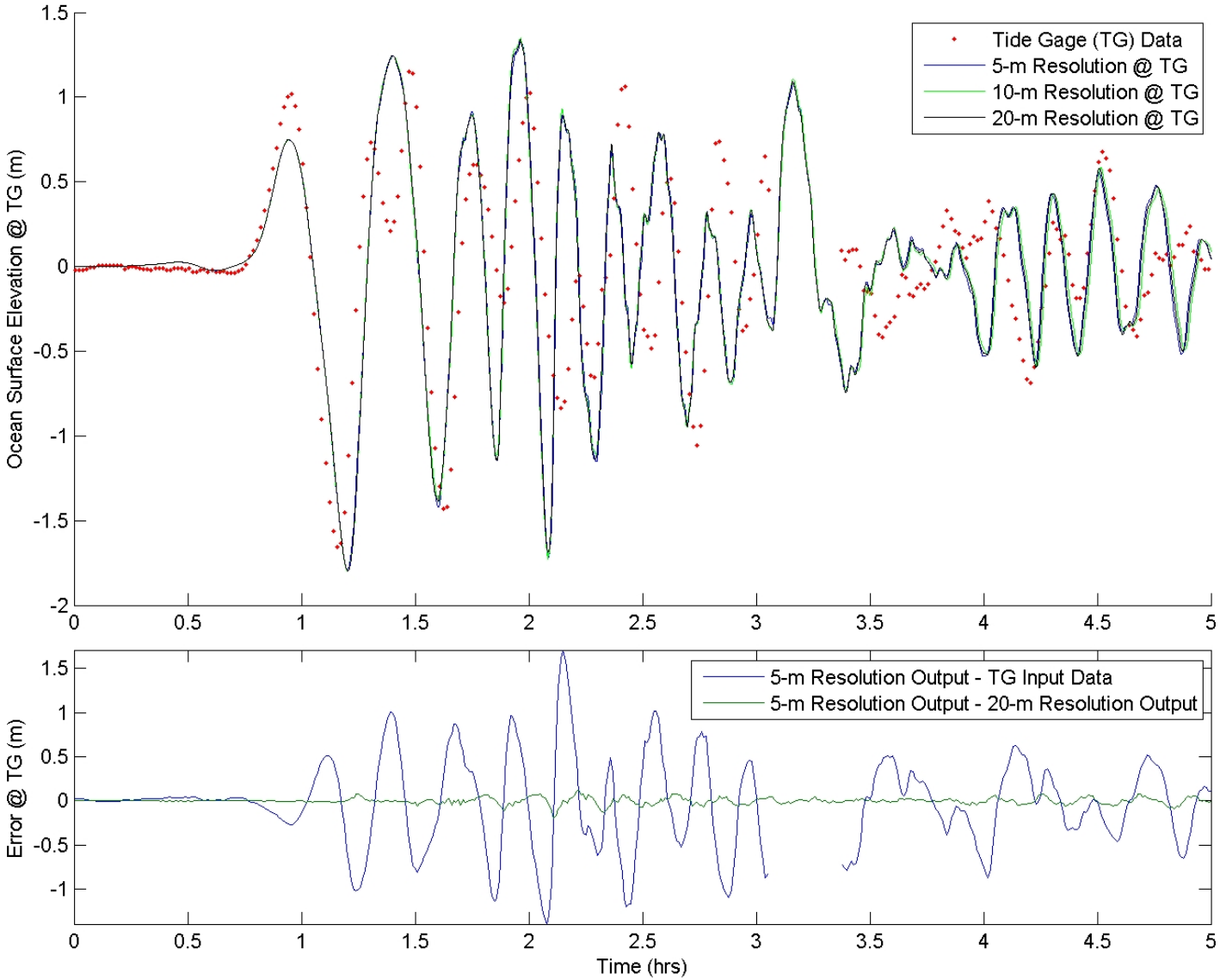
Benchmark #2 – Model Comparisons @ Control Point



All model runs are producing the same input wave (to within ~ 3cm)

Reproduction of the CP input data good (but not perfect)

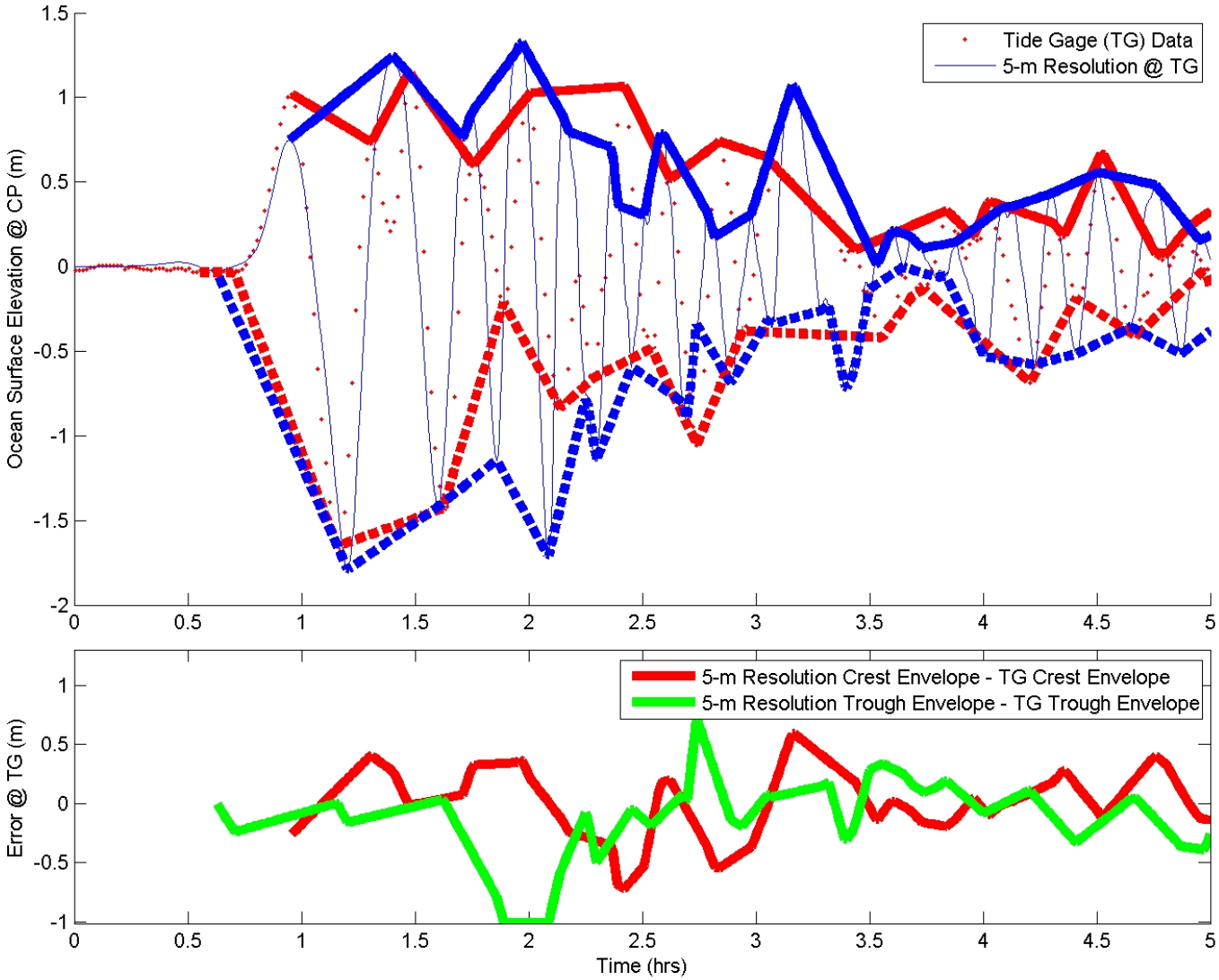
Benchmark #2 – Model Comparisons @ Tide Gage



Numerical convergence is excellent (to within a few cm)

OK agreement with data, but direct subtraction of time series may not be best approach…

Benchmark #2 – Model Comparisons @ Tide Gage



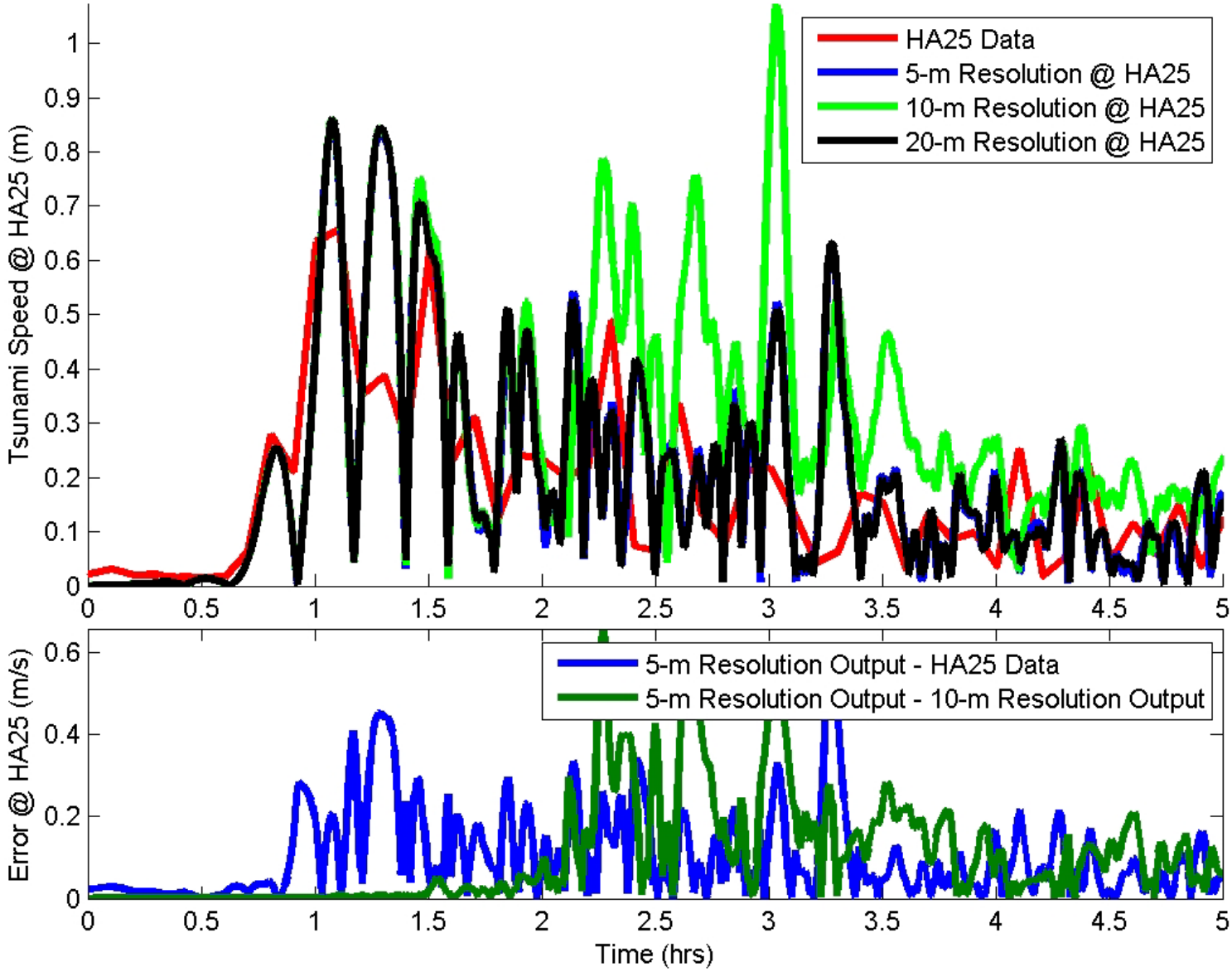
Comparing the envelopes of the time series gives a better picture

Agreement for first two waves excellent

Third/forth waves are OK

Good after

Benchmark #2 – Model Comparisons @ HA25

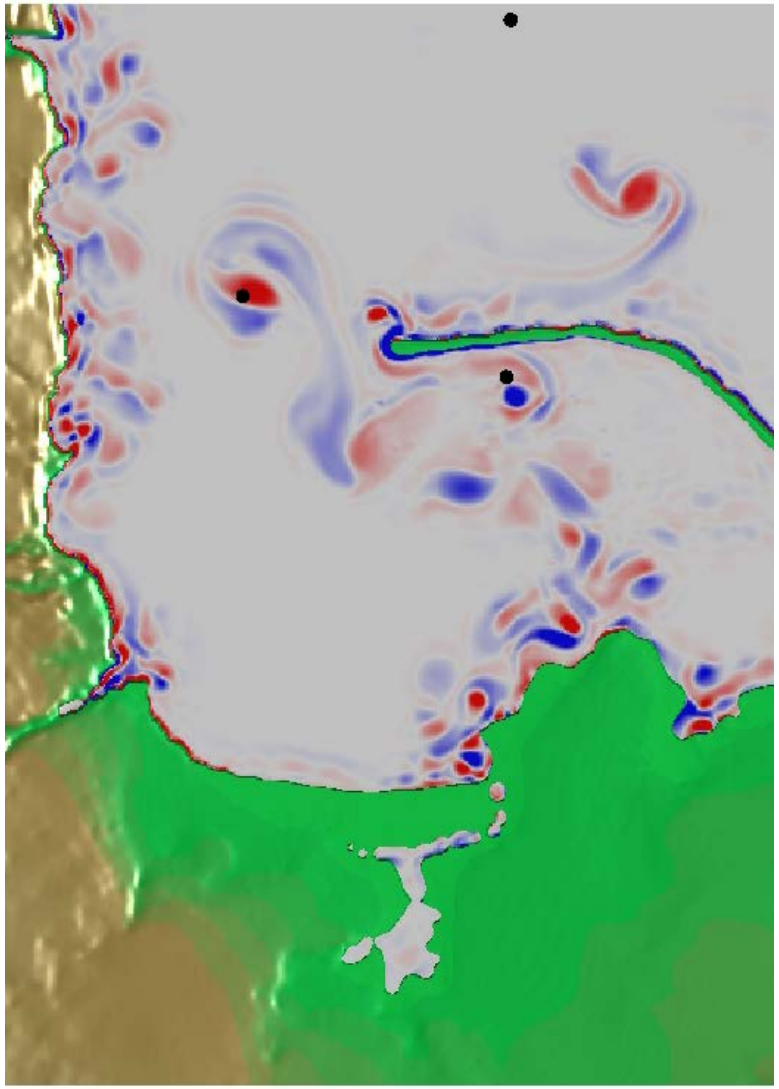


Numerical convergence for first few waves excellent

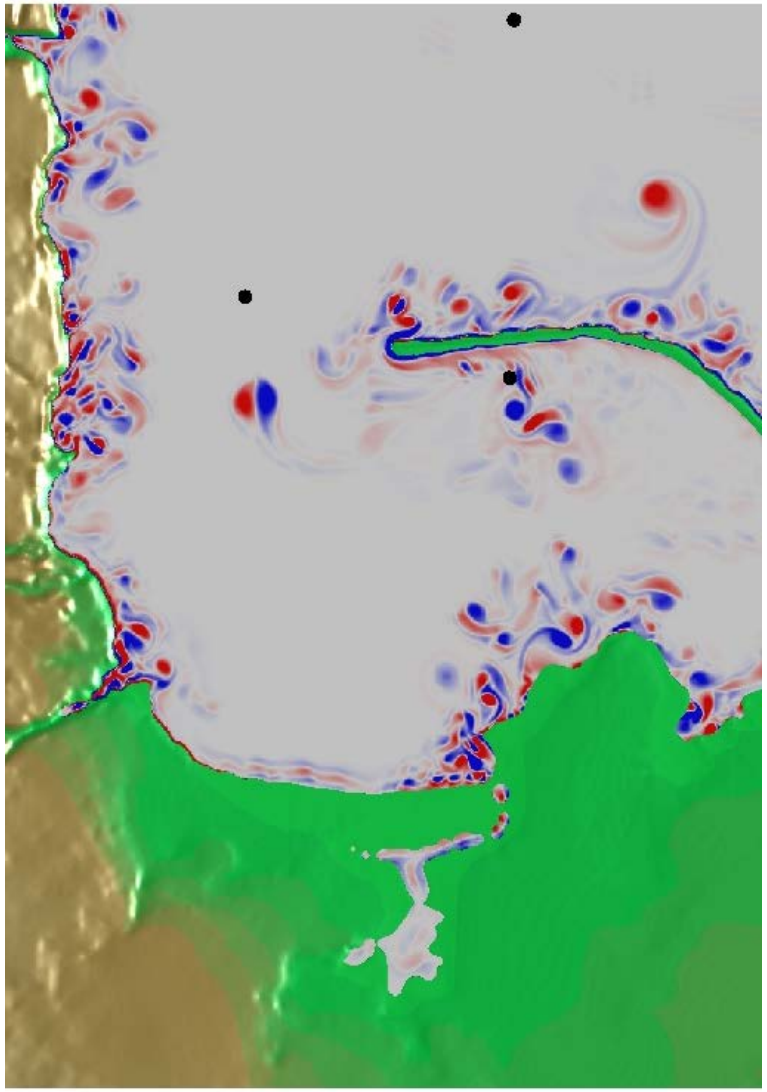
After that, large (100%) differences, inter-model variation similar to model error

Benchmark #2 – Model Comparisons @ HA25

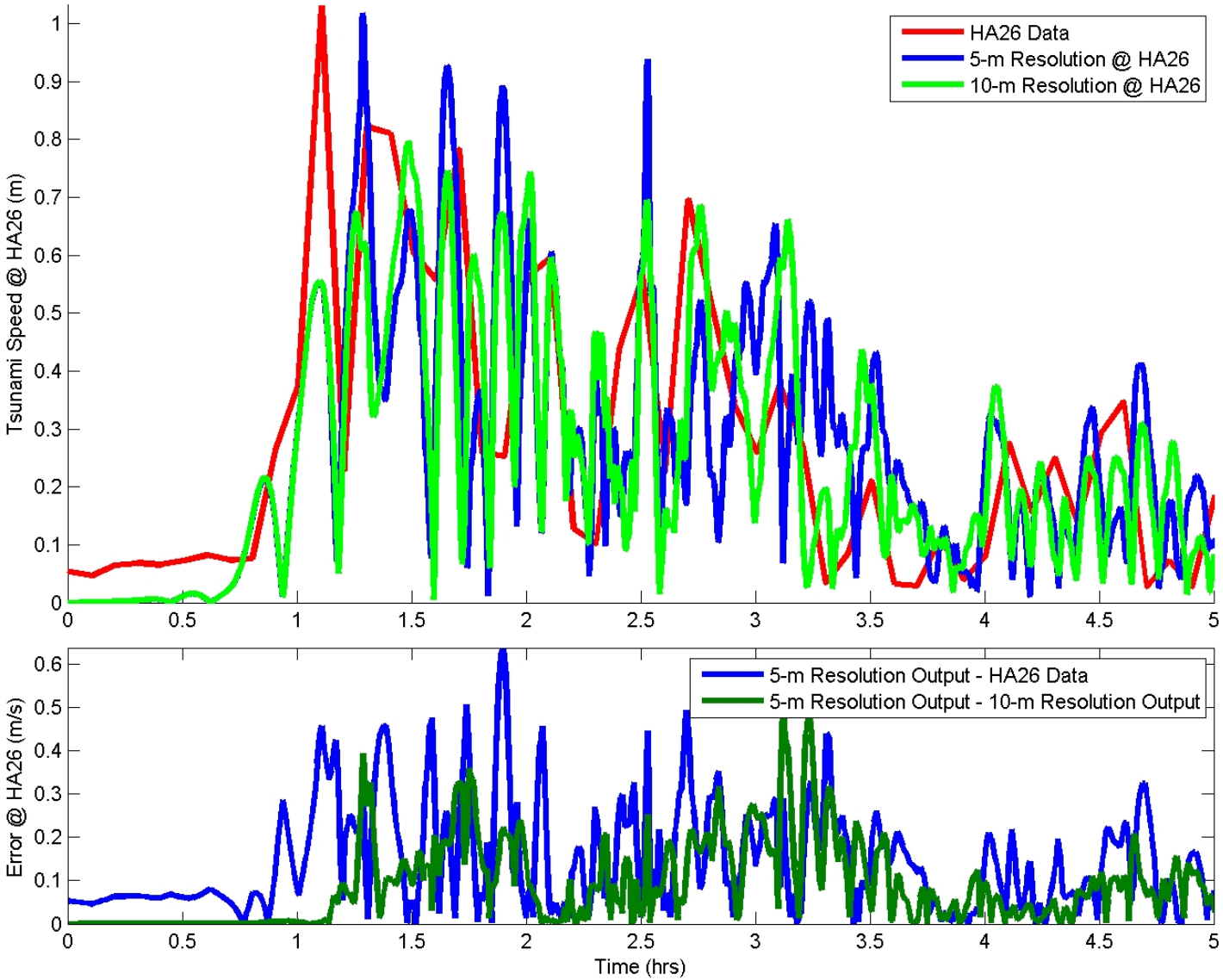
10 –m resolution



5 –m resolution



Benchmark #2 – Model Comparisons @ HA26

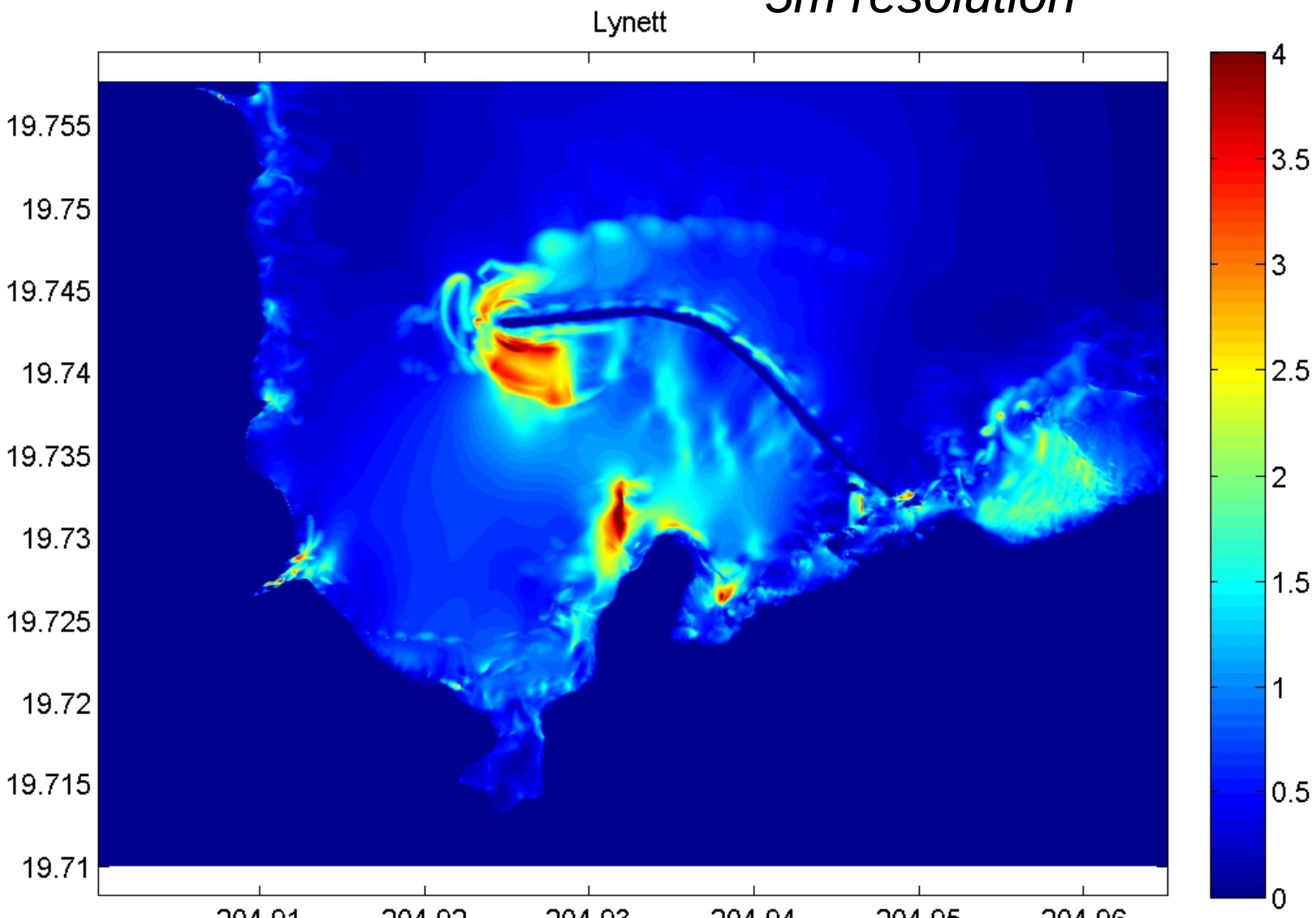


Numerical convergence only for first wave

After that, large (100%) differences, inter-model variation similar to model error

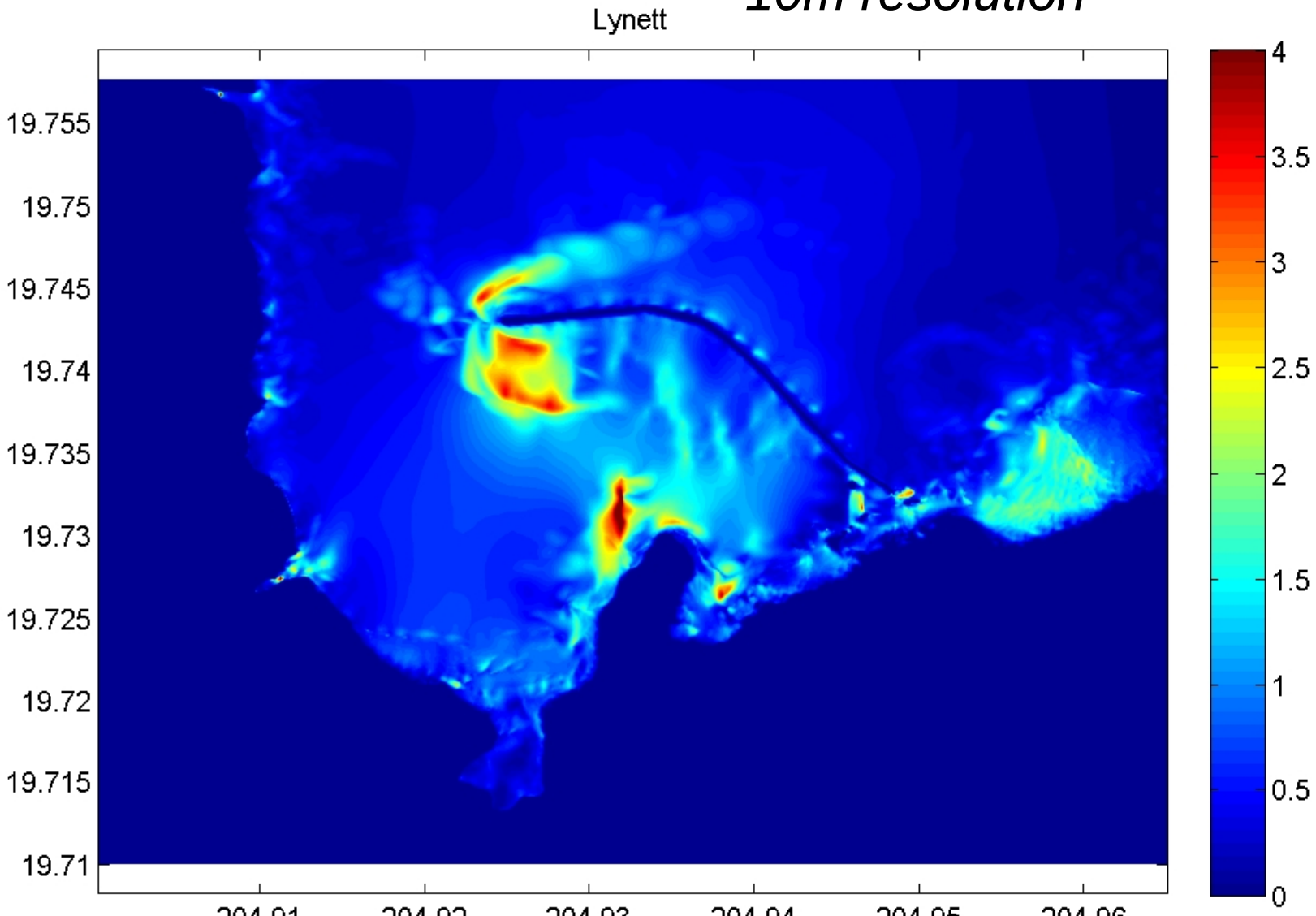
Benchmark #2 – Maximum Speed Comparisons

5m resolution



Benchmark #2 – Maximum Speed Comparisons

10m resolution



Conclusions

- Free surface elevation predictions and velocity predictions in regions not effected by eddies show convergence with grid resolutions of no less than 20 m
- In regions that are effected by eddies, there is NO numerical convergence in the deterministic sense down to a resolution of at least 5 m
- In these regions, variations and data errors are on the order of 50-100% of the flow speed

- ***In areas where currents are effected or controlled by eddies, what value does a deterministic simulation of currents have?***